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The boundless riches of God[1]

David D. Scott

The Revd. David D. Scott spent forty years working for the Church of Scotland mostly as a parish minister in Forth: St. Paul's, Logie Kirk, Bearsden: New Kilpatrick and Traprain. His first year of service was as a commissioned missionary teaching mathematics at Navrongo Secondary School, Ghana.

God has been described as 'infinite' in assorted confessional documents like the Scots Confession[2] and the Westminster Confession of Faith.[3] In the second chapter of the latter, "Of God, and of the Holy Trinity", proof texts support the statements that God is 'infinite in being and perfection' and 'his knowledge is infinite'.

Although the biblical witness never says that God is infinite with the clarity of our confessional statements, infinity is alluded to in many different places. There is the futile attempt to build a tower at Babel 'with its top in the heavens.' (Genesis 11:4)

This is followed by Jacob's ladder 'set up on the earth, the top of it reaching to heaven' (Genesis 28:12) and Isaiah's dramatic call where he sees the Lord 'sitting on a throne, high and lofty; and the hem of his robe filled the temple.' (Isaiah 6:1)

In the great Pauline hymns, the incarnation is celebrated in terms which reveal that the infinite character of God was made manifest in the finite humanity of Christ: 'For in him all the fullness of God was pleased to dwell.' (Colossians 1:20)

1. Intimations of infinity

One discipline in which infinity is normative is mathematics. Is it possible that mathematical enquiry into the character of infinity may cast some light on the infinite character of God? Three things suggest that it might.

Firstly, the quest for truth is something which is shared across all disciplines. In scientific enquiry, mathematics has been instrumental in affording greater understanding. Sometimes this has been totally unexpected.

For example, the invention of complex or imaginary numbers based on the square root of −1 was nothing more than an interesting game until it provided the mathematics to underpin the discovery of electromagnetism in the nineteenth century.

In his discussion of the relationship between science and theology, John Polkinghorne describes this as 'a deeper mystery'. It is the abstract nature of mathematics which surprises when it provides 'the key to unlocking the secrets of the physical universe'.[4]

Secondly, mathematics is a universal language which relates to everything in some way or another. Alain Badiou argues that 'Mathematics is the science of everything that is, grasped at its absolutely formal level'. He goes on to say:

> If, as regards what is, you want to know what it means to think only its being (i.e. not the fact that it's a tree, a pond, a man, but the fact that it *is*), the only way to do so is obviously to think purely formal structures, that is to say, structures indeterminate as to their physical characteristics.[5]

For him, mathematics is this science of ontology. If it is true and God is, then mathematics will surely be able to assist us in exploring the nature of God in his being if not enabling us in our finite nature to get closer to One who is 'infinite in being and perfection'.

Thirdly, in his book, "On Learned Ignorance", the medieval mathematician and theologian, Nicholas of Cusa (1401–1464), entitles his eleventh chapter, "That Mathematics Greatly Helps Us to Apprehend Different Divine Truths".[6]

He starts with finite mathematical figures with their attributes and relations and transfers them to corresponding infinite figures. He then applies the relations of the infinite figures to what he calls 'the infinite simple which is entirely independent even of every figure'.[7]

He concludes the process by adding, 'And then, as we labour in the dark of enigma, our ignorance will be taught incomprehensibly how we are able to think of the Most High more correctly and more truly.'[8] In this way, mathematics helps us to penetrate the mystery of God even if we will never fully comprehend him.

An early intimation of infinity was probably tied up with an exploration of the night sky and the Psalmist's sense of wonder about what is beyond our understanding (Psalm 8:3). Another may have been our experience of repetition. Some things, we say, go on forever.

A third may have been counting. 'One, two, many!' say the Piraha tribe of Brazil. 'To the mathematician, infinity *is* a reality.' said Einstein. 'In fact, mathematics could hardly exist without it, for it is inherent already in the counting numbers, which form the basis of practically all mathematics.'[9]

A fourth intimation may be found in a proof which was stated by Pythagoras (c. 570–c. 495 BC) and included in Euclid's *Elements*. It concerns the number of prime numbers. Using a startlingly simple proof, he proves that they never cease.

A fifth intimation may be found in the work of the Italian scientist, Galileo Galilei (1564–1642). In 1638, whilst under house arrest because of his support of Copernican cosmology, Galileo wrote, 'On Two Sciences' which included his observation concerning the number of natural numbers and their squares.[10]

He noticed that there is a one-to-one correspondence between every natural number and its square. For every n, you can write down n^2 where n is a member of the set of natural numbers. Although it looks as if the set of natural numbers should be bigger than the set containing all their squares, this is not the case.

Of course, it would be the case if we were just considering a finite subset of the set of natural numbers. But because of the one-to-one correspondence between the two infinite sets, something unusual is revealed. Galileo did not say in his book that the two sets are equal in number. Larger, smaller and equal don't seem to be appropriate here. The things that go on at infinity are alarming because they are counter-intuitive.

A sixth intimation of infinity may be found on the number line. As a consequence of his famous theorem concerning the lengths of a right-angled triangle, Pythagoras stumbled upon a length which could not be written down as a number, the square root of two, $\sqrt{2}$. He banned discussion of what was considered irrational. When Hippasus broke his oath of silence, he was assassinated.[11]

The Pythagorean School proved that √2 cannot be expressed as a fraction using the method *reductio ad absurdum*. It cannot happen because the decimal component stretches out to infinity without repeating. As it happens, there are two types of irrational numbers – algebraic like √2 and transcendental like π. It turns out that there are more irrational numbers on the number line and most of them are transcendental!

It was Bernhard Bolzano (1781–1848), a Czech priest, who discovered the density of the number line.[12]

Between any two numbers on the number line, there is an infinity of numbers. No matter how large or small the interval chosen, there are as many numbers in each. It looks as if infinity is not very far away.

2. Potential and actual infinity

The Greek word for infinity, απειρον made its first appearance in the philosophical work of Anaximander (c. 610–546 BC). It has to do with what is unlimited or boundless. It is the opposite of πέραs which is all about limits and bounds. In addition, απειρον also had connotations of the chaotic or irrational.[13]

The paradoxes of Zeno (c. 490–c. 430 BC) illustrated the contrast between the finite and the infinite where disorder prevailed and Achilles and the tortoise couldn't complete the race because an infinite series had been created! The Pythagoreans rejected infinity because it had nothing to do with the real world. And with Aristotle (384–322 BC), a distinction was made between actual and potential infinity.

'[H]e altered the concept of infinity to mean an unending and incomplete process' writes Robert John Russell, '… something capable of being endlessly divided or added to, but never fully actualized as infinite.' This conception continued in Western thought without serious challenge until the nineteenth century.[14]

This potential infinity was anticipated by Eudoxus (c. 408–c. 355 BC) who developed 'the method of exhaustion, devised to compute areas and volumes'.[15] It divided up magnitudes into quantities which were as small as he liked and did not depend on the existence of infinitely many or infinitely small for its success. Almost a century later, Archimedes followed with his use of infinitesimal quantities.

The fear of the infinite had to do with disorder rather than order, irrationality rather than the rational. It reappeared in an extraordinary way in seventeenth-century Italy where the Jesuits declared war on those who espoused the mathematics of the infinitely small. They valued the order of Euclid's *Elements* and feared the paradoxes inherent in the infinite.

'One approach emphasised the purity of mathematics, the other emphasised practical results;' writes Amir Alexander, 'one approach insisted on absolute perfect order, the other was willing to coexist with ambiguities and uncertainties.' It was an aspect of mathematics which anticipated the calculus of Leibniz and Newton and a deeper understanding of the modern world.[16]

2.1 Euclid

Paradoxically, the Euclid which the Jesuits thought was such a worthy tool to impose order on a disorderly world was not devoid of uncertainty. Euclid was the first to acknowledge this when he questioned the validity of his fifth axiom, the Parallel Postulate.

Euclid was troubled by it for several reasons. It was more complex than the other axioms. It seemed less evident. He wondered whether or not it could be a theorem with a proof derived from the other axioms.[17]

Mathematicians like Ptolemy tried in vain to realise this.

The problem arose not least because of its hidden reference to infinity. If you consider a finite space and draw a line and a point not on that line then you can draw any number of lines through the point which will be parallel to the original line within that finite space. For they will never meet!

For over two thousand years, mathematicians accepted the postulate and the existence of parallel lines without question until the early nineteenth century when mathematicians decided to play about with his understanding of parallel lines.

Instead, of building their mathematics on Euclid's understanding of parallel lines, they created a universe in which there were no parallel lines. What would happen then? It was Bernhard Riemann who developed this elliptical geometry. It was more like a puzzle or a game than a reflection of reality!

The sceptics had a field day. The mathematicians enjoyed their discoveries. Nothing real appeared to be reflected in what they were doing until the early twentieth century! A brilliant, young physicist was reordering our understanding of the universe. Instead of three dimensions, he was looking at four – space/time.

Instead of straight lines, he was looking at curved lines. He needed some mathematics to work out what was going on. The mathematics of Euclid was all three dimensional. But what about Riemann's elliptical mathematics where there are no parallel lines? Amazingly, it was this unusual way of looking at the universe which provided Albert Einstein with the mathematics for his great 'Theory of Relativity'.

With the existence of these new geometries, it became apparent that Euclidean geometry was a theory which was consistent in terms of the Euclidean plane. It had nothing to do with the natural world for it was composed in quite a different way! It raised important questions about the nature of mathematics and its relationship with reality.

2.2 Georg Cantor

Up until the time of Georg Cantor (1845–1918), mathematicians had followed Aristotle's lead in working with potential infinity rather than actual infinity. They were afraid that it would be full of antinomies like the one which Galileo had revealed. Cantor made it his life's work.

'The fear of infinity is a form of short-sightedness that destroys the possibility of seeing the actual infinite,' wrote Cantor, 'even though it in its highest form has created and sustains us, and in its secondary transfinite forms occurs all around us and even inhabits our minds.'[18]

Cantor created a new kind of number – transfinites. The transfinite was a measure of the number of elements in an infinite set. He labelled these numbers using the Hebrew symbol aleph with a subscript 0 for the infinite number of integers.

Because there is a one-to-one correspondence between the integers and the rational numbers and also the algebraic numbers, these sets have the same transfinite number. This isn't true for the real numbers which are not countable. Cantor created another transfinite for them – aleph with subscript 1.

Because the set of subsets of a particular set is larger than the set itself, Cantor was able to show that this set always had a transfinite number greater than the one for the set itself. He then considered the set of all transfinite sets. He called this Ω or absolute infinity.[19]

There is what Cantor called 'an unresolvable antinomy' here. If you gather up all the sets in the universe into one set and call it *Ananta*, Sanskrit for 'infinity', the transfinite for this set is aleph-infinity. If it is the set of all possible sets then it must contain its own power set, the set of all subsets in *Ananta*.

This leads to a contradiction. *Ananta* is strictly smaller than its own power set. But Ananta is also strictly bigger because its power set is a subset of *Ananta* and subsets are always strictly smaller than the set itself.[20]

Cantor resolved this inherent contradiction by claiming that Ω or absolute infinity 'cannot be an object of quantitative, discursive, rational operation' writes Wolfgang Achtner. 'Furthermore, it cannot be recognised, for it can only be accepted without any further discursive rational activity and logical discernment.'[21] Cantor claimed that this was God!

He claimed that the Absolute was beyond our reach. Writing to an English mathematician, Grace Chisholm Young, he argued that he had never proceeded from any *Genus supremum* because there was no *Genus supremum* of the actual infinite:

> What surpasses all that is finite and transfinite is no 'Genus'; it is the single, completely individual unity in which everything is included, which includes the *Absolute*, incomprehensible to the human understanding. This is the *Actus Purissimus*, which by many is called *God*.[22]

As it happened, Cantor was genuinely surprised by his discoveries. 'I see it, but I do not believe it' he said. A contemporary mathematician, David Hilbert, praised him for his extraordinary work adding, 'no one shall expel us from the paradise which Cantor has created for us.'[23]

2.3 Kurt Gödel

One of the problems which Cantor struggled to solve in his lifetime may have contributed to the mental illness which ultimately ended his life. It was called the Continuum Hypothesis and remained unsolved until 1963.

Cantor had identified two transfinites – aleph-zero and aleph-one. The former related to the set of integers which were infinite but countable. The latter related to the real numbers which were infinite but uncountable. They could be identified with the points on a straight line, the continuum.

Cantor wondered whether there was an infinity bigger than the infinity created by the integers but smaller than the continuum. As it happened, the solution was surprising. The hypothesis was both true and false depending on initial assumptions.

'It […] is independent of the axioms of set theory' writes Eli Maor, 'it can be regarded as an additional axiom which we are free to accept or reject.'[24]

New insights like these were discovered by the Austrian mathematician and great friend of Einstein, Kurt Gödel. They were both professors at Princeton and Einstein once famously said 'that he came to the Institute merely […] to have the privilege of walking home with Gödel.'[25]

On 7 October 1930, Gödel was giving a short, twenty-minute paper at a conference in Königsberg. Although it attracted very little attention at the time, it eventually created shockwaves throughout the mathematical world.

His theme was completeness. It is summarised in what are called the First and Second Incompleteness Theorems:

> 1. Any consistent formal system *S* within which a certain amount of elementary arithmetic can be carried out is incomplete with regard to statements of elementary arithmetic: there are such statements which can neither be proved nor disproved in *S*.
> 2. For any consistent formal system *S* within which a certain amount of elementary arithmetic can be carried out, the consistency of *S* cannot be proved in *S* itself.[26]

Of course, it may be proved in some alternative system but not in *S* itself. In other words, the quest for establishing a formal system which would include all mathematical thought is a futile one. There will also be some mathematics which will exist outwith the system.

In addition, there will be limits put upon the mathematics which can be accomplished in such a formalised system. There will be statements which although true cannot be proved nor disproved. The world of mathematics is bigger than our human systems.

Three things follow. Gödel's theorems proclaim what Rebecca Goldstein calls 'the robustness of the mathematical notion of infinity'.[27] There is no way of eliminating the concept nor of reducing the concept so that it can sit comfortably within a formalised system. It will always elude us.

John Polkinghorne argues that like the scientist who has to put his faith in the intelligibility of the universe to unravel the truth, Gödel's work on incompleteness reveals that the study of pure mathematics is also 'an act of intellectual daring',[28] trusting in what cannot be proved within the system.

According to Sir Roger Penrose, Gödel also said something important about the philosophy of the mind: 'his results […] established that human understanding and insight cannot be reduced to any set of rules.'[29]

Human beings are not machines like computers which are based on formal finite systems run on algorithms.

Whilst it is natural for human beings to want to impose order on the universe, the world of mathematics illustrates that there is more to the cosmos than our systems and structures no matter how logical they appear to be. We will always have to make room for the infinite and this brings with it paradox which Gödel specifically utilised to great effect in his extraordinary proof!

3. Creative tensions

In our thinking about the infinite, some tensions have emerged which are not readily resolved. Perhaps these are the very things which enable us to explore further, embrace the paradoxical and the things which are counter-intuitive in our quest for the truth.

3.1 the finite and the infinite

What is finite is limited and two limitations come to mind. The first is very small. Contemporary science has made it clear that there is a point where matter starts to become indivisible. This is the Planck length defined as 1.6×10^{-35} metres!

The second is very big – the Bekenstein bound[30] which sets a limit on the amount of information which can be generated in a finite section of space. It puts a limit on the number of thoughts which can be stored in the brain at any one time. For an average brain, it is approximately 10^{10}.

If God is infinite then the generation of our finite thoughts will never be sufficient to comprehend God. As the psalmist says, 'How weighty to me are your thoughts, O God! How vast is the sum of them!' (Psalm 139:17)

For more than two thousand years, mathematicians viewed infinity negatively. Finite things were rational, orderly, unambiguous. The infinite produced chaos, irrationality, paradox and contradiction. It was Georg Cantor who viewed infinity more positively. He not only enabled us to see the character of the infinite but led us to a place where we could catch a glimpse of the living God in a finite human being.

Jesus is described as fully human and fully divine. This contradicts our understanding of the world and reveals a gospel which is counter-intuitive. The last shall be first, the first last. The way to greatness is to forget self. Those who are prized are the foolish and the weak. The incarnation is radical and paradoxical.

3.2 theory and reality

In our understanding of the universe, there is much theoretical explanation but we live with the uncertainty of two theorems which cannot be reconciled as yet – the Theory of General Relativity which explores the macroscopic picture and Quantum Mechanics which explores the microscopic.

In investigating the physical world from a scientific perspective, John Polkinghorne considers the method of critical realism.[31] The realism acknowledges the positive relationship which exists between scientific enquiry and the real world.

The critical dimension acknowledges the process whereby experiments are made, theories are drawn up to explain them and when other information contradicts the theory, more experiments follow and there is further theoretical development.

He argues that 'scientific discovery requires the boldness of provisional commitment to a point of view, while remaining aware that this may require subsequent modification in the light of further experience'.[32]

In mathematics, it is not experiment which drives the discipline but proof. It was the fruit of Greek culture – the logic of the philosophers and the stories dramatized in the amphitheatre. It is exemplified in Euclid's *Elements* where logical tales are told and elegant pictures drawn to compel belief.

By this method, mathematicians are able to prove that some things are impossible like squaring the circle which defied proof for so long despite many valiant efforts because it was based upon the irrationality of π.

The proof reveals the truth. Those who do not accept it are challenged to disprove it. But once they are proved wrong, it is accepted by the whole community. To this extent it is a democratic way to proceed and to search for the truth.

In the revelation of God contained in Scripture, we cannot command the assent of people by a mathematical proof. Theology is more like scientific enquiry where new insights challenge traditional understanding and the realities of the contemporary world shape our priorities and perspectives.

In the Scots Confession, the five Johns who wrote it were keen to secure universal approval for their understanding of the Scriptures and the things of God. In the preface, they wrote:

> If any man will note in our Confession any chapter or sentence contrary to God's Holy Word, that it would please him of his gentleness and for Christian charity's sake to inform us of it in writing; and we, upon our honour, do promise him that by God's grace we shall give him satisfaction from the mouth of God, that is, from Holy Scripture, or else we shall alter whatever he can prove to be wrong.[33]

In this way, Polkinghorne's critical realism serves to secure a provisional commitment to a particular point of view with an openness to change it if necessary after critical reflection. Although there is no certain proof as in mathematics, there is an attractive democracy created where the provisional gives way to a more inclusive community of believers.

3.3 discovery and invention

There are two ways of understanding the work of the mathematician. There are the realists, the ones who see mathematics as integrated into the fabric of the universe and the very language with which the universe is understood. Their mathematics was like a voyage of discovery.

In his beautiful autobiography, *A Mathematician's Apology*, the Oxford mathematician G. H. Hardy makes his position very clear when he writes, 'I believe that mathematical reality lies outside us, that our function is to discover or *observe* it, and that the theorems which we prove, and which we describe grandiloquently as our 'creations', are simply our notes of our observations.'[34]

Hardy and others talk about mathematics as being beautiful. Beauty may seem an unusual quality to look for in numbers, signs and symbols but how else would you describe Descartes' amazing transformation of geometry into algebra through what we now call Cartesian co-ordinates and the transformation of physics into geometry in Einstein's Theory of Relativity?

The latter relied on the use of an alternative geometry. When these geometries were unfolded, the mathematical world began to be persuaded that this was more invention rather than discovery. Euclid's geometry was reviewed and seen as one geometry among many. As Mario Livio asked after Plato, 'if God ever geometrizes […] which of these many geometries does the divine practice?'[35]

This encouraged the formalists who saw mathematics as invention. One of the leading advocates was the German mathematician David Hilbert (1862–1943) who said, 'Mathematics is a game played according to certain simple rules with meaningless marks on paper.'[36] This language game was played for its own sake. Having a relationship with reality was not a necessary prerequisite!

Hilbert attempted to formalise mathematics but his attempts were seriously challenged by Gödel's two theorems of incompleteness. There were limits to invention. Mathematics seemed to have a mind of its own. It defied being constrained by axiomatic structures. It preferred paradox and contradiction. It was even bigger than expected!

It is difficult to unravel these two threads – realism and formalism. Sometimes mathematics looks invented like the alternative geometries but in time they look more like discoveries which help scientists like Einstein to understand the universe. Perhaps it is more to do with perspective. Invention has a big ego. Discovery plays the humbler part. The one is like theology, the other revelation.

Revelation is given. Nothing can be added to it nor taken away. Everyone is free to experience it and to think about whatever it means. There may be a theological construct put upon it but it requires a democracy of belief to survive. It is a human invention which must be offered in humility for it may not be able to contain the whole revelation. There is always more to discover in the counter-intuition.

The people of Israel were appalled at what Isaiah was saying about God's servant. His humiliation contradicted their understanding of how things should be. 'He had no form or majesty that we should look at him, nothing in his appearance that we should desire him' (Isaiah 53:2b). And yet he is the one chosen to bear our infirmities and by his bruises we are healed. This is a radical revelation but like the non-Euclidean geometry had to wait until the time was right!

4. Mathematics and theology

Towards the end of the fourteenth century, an anonymous text appeared for the benefit of those who wanted to become contemplatives. Instead of focusing on the humanity of God made man, it focuses on the divine nature of God through a process of unknowing.

In *The Cloud of Unknowing*, the teacher makes it clear that their initial experience will be of darkness. It is 'an absence of knowing' which is like a cloud between the student and God.[37]

The teaching is austere. When the student asks by what means he is to achieve this work of contemplation, the teacher beseeches God to teach the student himself. 'For I would have you know well that I cannot tell you; and that is no surprise, because it is the work of God alone …'.[38]

This is the *via negativa*, an example of apophatic theology, setting our gaze beyond words to the source of our being. It's what Denys Turner describes as 'that speech about God which is the failure of speech'.[39] Notwithstanding the austerity of the teaching, it is enfolded in a lot of words.

There is an obvious paradox inherent in this approach. The apophatic is supplemented by the kataphatic. A negative way cannot be pursued unless something is negated. To this extent, what is negated is in itself vital to the whole process. The two approaches are related. The paradox celebrates it.

According to Robert John Russell, Founder and Director of the Centre for Theology and the Natural Sciences, Berkeley, California, the apophatic and the kataphatic are brought together through Cantor's Reflection Principle.[40]

In a very ingenious way, Cantor was able to prove that the Absolute Infinity, Ω, lying beyond the transfinites and described by the mathematician in the above letter as 'incomprehensible to the human understanding' was both conceivable and inconceivable.

Russell argues that the transfinites are infinities whose properties can be defined and their identities distinguished. The Absolute Infinity shares properties with each of the transfinites which are clearly disclosed to us. To this extent, the Absolute Infinity is comprehensible.

But none of the properties of the Absolute Infinity is unique to itself and therefore it cannot be uniquely described. For this reason it is incomprehensible. What we know of Absolute Infinity is only partial and that partial knowledge is harvested from what we know of the transfinites.

What makes Absolute Infinity totally unique is never disclosed to us. This is the part which is hidden from view. Russell goes on to use a metaphor to clarify the argument: 'it is as though the transfinites form an endless veil surrounding Absolute Infinity.'[41]

What we know about Absolute Infinity is in the veil. What is behind the veil is forever beyond our ken. However, we are able to learn more and more about the Absolute Infinity as we learn more and more about the transfinites.

Moving from mathematics to theology, Russell describes God as 'Absolute Mystery, the ineffable source of knowledge, wisdom and existence lying forever beyond human comprehension'.[42] Alongside this is the Christian understanding that this incomprehensible God chose to make himself known to us as our Creator and Redeemer.

He argues that this theological construct is analogous to Cantor's Reflection Principle. 'The God who we know as Creator and Redeemer is inherently incomprehensible Mystery. The God who is known through special revelation (word and scripture) and general revelation (nature) is known as unknowable.'[43]

He recognises an analogy between the mathematics of infinity and the theology of revelation. Just as the inconceivable nature of the Absolute Infinity is secured by its reflection in the transfinites so the mystery of God is secured by the revelation of God in our lives and the universe. He concludes by suggesting that revelation is 'the veil that discloses'.[44]

Russell then teams up with Wolfhart Pannenberg who uses the concept of infinity to underpin his 'Doctrine of God'. It is more than an opposite to the finite. If that is all it was it would be finite itself for its definition is entirely bound up with what is finite.

He explores the holiness of God which provides a sharp distinction between the finite and the infinite. In his holiness, God is quite distinct from everything else. This is the nature of his being. But in his wisdom, God chose not to remain apart from the profane world. Instead he chose to embrace it.

In this way, God not only opposed what is profane but also transformed it. As St Paul says, 'Through Christ, God was pleased to reconcile to himself all things, whether on earth or in heaven, by making peace through the blood of his cross.' (Colossians 1:20)

At the very least, an exploration of mathematical infinity provides useful analogies to explore the God who is 'infinite in being and perfection'. This is evident in the paradoxical character of the infinite. It is a place which is counter-intuitive, full of ambiguity and surprise.

Contemporary mathematics has illustrated that some things cannot be proved and some aspects of the physical world are undecidable.[45] There are areas of knowledge and understanding which the human mind will never be able to penetrate. Nowhere is this more true than in the infinite God.

Cantor equated a mathematical result with God. Russell with the help of Pannenberg's theology used the mathematician's conclusion to provide a powerful analogy for the knowability of God. For Cantor, mathematics as a constituent part of the universe provides an opening into natural theology.

Whether by analogy or by natural theology, an exploration of mathematical infinity sharpens our perceptions through this cross-fertilization, takes us to a place of new horizons and deepens our sense of wonder in the God who has created a universe which can only ultimately be understood in a mathematical way.

Notes

[1] Adapted from Ephesians 3:8.
[2] *The Scots Confession of 1560*, ed. G. D. Henderson; trans. James Bulloch (Edinburgh: Saint Andrew Press, 1991), 5.
[3] *The Confession of Faith […]* (Edinburgh: Blackwood, 1969), 6.
[4] John Polkinghorne, *Science and Religion in Quest of Truth* (London: SPCK, 2011), 72.
[5] Alan Baidou with Gilles Haéri, *In Praise of Mathematics*, trans. Susan Spitzer (Malden, Mass.: Polity Press, 2016), 65.
[6] *Nicholas of Cusa: Selected Spiritual Writings*, trans. H. Lawrence Bond (New York: Paulist Press, 1997), 100.
[7] Ibid., 102.
[8] Ibid.
[9] Eli Maor, *To Infinity and Beyond: A Cultural History of the Infinite* (Princeton N.J.: Princeton University Press, 2017), 233.
[10] Leonard Mlodinow, *Euclid's Window: The Story of Geometry from Parallel Lines to Hyperspace* (London: Penguin, 2002), 51.
[11] Ibid., 26.
[12] Amir D. Aczel, *The Mystery of the Aleph: Mathematics, the Kabbalah, and the Search for Infinity* (New York: Washington Square Press, 2000), 57–63.
[13] Graham Oppy, "God and Infinity: Directions for Future Research" in *Infinity: New Research Frontiers*, edited by Michael Heller and W. Hugh Woodin (Cambridge: Cambridge University Press, 2011), 234.
[14] Robert John Russell, "God and Infinity: Theological Insights from Cantor's Mathematics", in ibid., 278.
[15] Aczel, *Mystery of the Aleph*, 22.
[16] Amir Alexander, *Infinitesimal: How a Dangerous Mathematical Theory Shaped the Modern World* (Richmond: Oneworld, 2014), 170.
[17] David Berlinski, *The King of Infinite Space: Euclid and His Elements* (New York: Basic Books, 2013), 54.
[18] Quoted in Rudy Rucker, "Introduction" in Heller and Woodin, eds., *Infinity: New Research Frontiers*, 4.
[19] Wolfgang Achtner, "Infinity as a Transformative Concept in Science and Theology", in ibid., 39 f.

[20] H. Chris Ransford, *God and the Mathematics of Infinity: What Irreducible Mathematics Says About Godhood* (Stuttgart: ibidem-Verlag, 2017), 40.
[21] Achtner, "Infinity as a Transformative Concept", 41.
[22] Aczel, *Mystery of the Aleph*, 189.
[23] Quoted in Marcus Du Sautoy, *What We Cannot Know: Explorations at the Edge of Knowledge* (London: 4th Estate, 2016), 398.
[24] Maor, *To Infinity and Beyond*, 65.
[25] Rebecca Goldstein, *Incompleteness: The Proof and Paradox of Kurt Gödel* (New York: W. W. Norton, 2005), 33.
[26] Mario Livio, *Is God a Mathematician?* (New York: Simon & Schuster, 2009), 196.
[27] Goldstein, *Incompleteness*, 186.
[28] John Polkinghorne, *Belief in God in an Age of Science* (New Haven: Yale University Press, 1998), 132.
[29] Goldstein, *Incompleteness*, 201 f.
[30] Ransford, *God and the Mathematics of Infinity*, 42.
[31] Polkinghorne, *Science and Religion in Quest for Truth*, 11.
[32] Ibid., 9.
[33] *Scots Confession*, 3 f.
[34] G. H. Hardy, *A Mathematician's Apology* (Cambridge: Cambridge University Press, 1998), 123 f.
[35] Livio, *Is God a Mathematician?*, 162.
[36] Goldstein, *Incompleteness*, 136.
[37] *The Cloud of Unknowing and Other Works*, translated with an introduction and notes by A. C. Spearing (London, Penguin Books, 2001), 26.
[38] Ibid., 56.
[39] Quoted in J. P. Williams, *Seeking the God Beyond: A Beginner's Guide to Christian Apophatic Spirituality* (London: SCM Press, 2018), xix.
[40] Russell, "God and Infinity", 282 ff.
[41] Ibid., 284.
[42] Ibid.
[43] Ibid., 284 f.
[44] Ibid., 285.
[45] Toby S. Cubbitt, David Perez-Garcia and Michael Wolf, "The Un(solv)-able Problem", *Scientific American* 319:4 (October 2018): 28–37.