Material Implication and Indicative Conditionals

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Introduction and Definitions

It has often been asked whether the truth-function known as material implication correctly accounts for conditionals in the indicative mood. After defining material implication and indicative conditionals (hereafter just "conditionals"), I will discuss why I believe the former does not always account for the latter. Defences for a material interpretation of conditionals by H. P. Grice and Frank Jackson will then be given.

A function is analogous to a machine which outputs something when something is input. The inputs and outputs of truth-functions are truth values: "true" or "false". The symbol for material implication ('⊃') is thus formally defined: if the sentence before it (the antecedent) is true and the sentence after it (the consequent) is false, then the material implication is false; otherwise it is true.

Conditionals are a complex sentence form; they are made up of sentences and can be either true or false (but not both). If *A* and *B* are any sentences, then "If *A*, then *B*" is the conditional form. The previous sentence is also a conditional (*A* and *B* can be complex sentences, like "The flag is raised and somebody is dead.") As with material implication, *A* is the antecedent and *B* is the consequent.

Conditionals with synthetic antecedents and consequents will be considered, rather than conditionals with analytic antecedents or consequents. The subject in a synthetic sentence – like "the flag" in the sentence "The flag is raised" – does not somehow contain the predicate (here "is raised"). Contrast this with the analytic sentence "The white swan is white." Since this cannot be false, we cannot speak of "If the white swan is white, then the white swan is white" having a false antecedent or consequent, which is crucial.

Material Implication does not Necessarily Express Conditionals

Does material implication correctly account for, say, "If the flag is raised, then somebody is dead"? The question is whether the sentence is false when "The flag is raised" is true and "Somebody is dead" is false, but true otherwise.

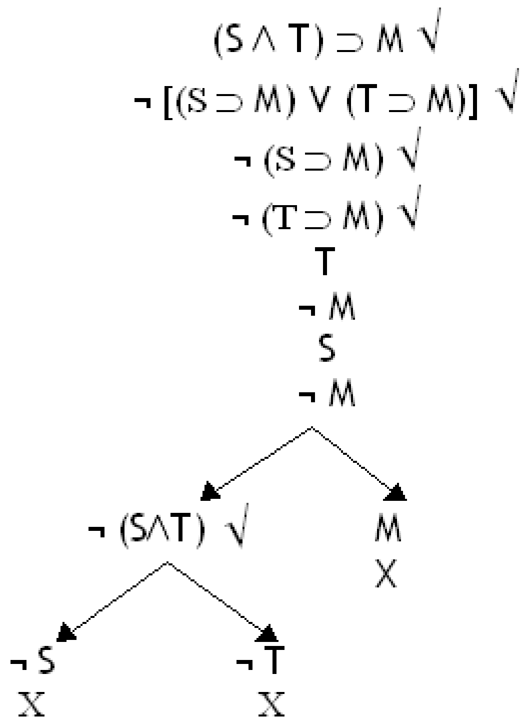
First of all, a speaker of the sentence is not necessarily saying anything about "The flag is raised" being false or anything consequent on its falsity. They are not doing so explicitly in any case. The assertion may just be that a dead person is a necessary and sufficient condition for a raised flag. The sentence is true if both the antecedent and consequent are true; the sentence is false if the antecedent is true and the consequent is false. That is all.

Secondly, conditionals can be used within a non-formal language for different purposes. They do not always operate under the same truth conditions. There are circumstances in which the truth conditions of the sentence "If the flag is raised, then somebody is dead" are more numerous than the above: a person may say it within the context of a military base, implying strongly that if the flag is not raised then nobody is dead. If this occurs, then the sentence (its suggestion strictly speaking) is true. It is, however, difficult to imagine a case where the sentence is true when the antecedent is false and the consequent is true.

There is a popular counterexample to the material account of conditionals by William S. Cooper. Suppose there is a motor hooked up to two switches (S and T) and that the only information we are given is expressed by the sentence "If S and T are presently thrown, then the motor starts."

This sentence is formalized, on the material interpretation, as *(S ∧ T) ⊃ M*. Throwing both switches is a sufficient condition for the motor starting, but it is unknown whether the motor starts if either switch is thrown independently. The sentence "'It is the case for one or other of the switches that if that switch is thrown (independent of whether the other is) that the motor will start" (formalized on the material interpretation as *[(S ⊃ M) v (T ⊃ M)]*) can be false.

But in normal classical logic, the latter cannot be false if the former is true:



Defences of the Material Account of Conditionals

The traditional view nevertheless posits conditionals as accounted for by material implication. One argument for this view (**A1**) relies on the implicational relationship between disjunctions (complex propositions of the form "either A or B") and conditionals:

Assumptions	**Formulae**	**Justification**
1	(1) E	Assumption
2	(2) D	Assumption
1,2	(3) L	1,2
1,2	(4) ¬ R v S	3 NC formalization
1,2	(5) R⊃S	4 Implication

("D" = "if the flag is raised, then somebody has died"; "E" = "propositions of the form 'if A, then B' are equivalent to propositions of the form 'either not A or B'"; "L" = "either the flag is not raised or somebody has died"; "R" = "the flag is raised"; "S" = "somebody has died.")

Grice and Jackson: The Counterexamples Cannot be Asserted

H. P. Grice accepts the material interpretation of conditionals. He therefore considers statements such as the following to be paradoxes: no proposition can imply (as the antecedent of a conditional) an arbitrary consequent by being falsified; yet '$P \supset Q$' cannot be false if '$\neg P$' is true. His response to purported counterexamples is to introduce a distinction between two properties of propositions: appropriateness for conversation and truth. Neither implies the other.

Whether or not a proposition should be asserted is determined by certain maxims. The maxims of quality and quantity particularly ensure the cooperation of language users. The maxim of quality is a requirement for propositions to be true and justified. The maxim of quantity requires the contribution of the speaker to be sufficiently informative but not more informative than is necessary (Grice is uncertain about the latter point). Suggestions follow from conversation when the maxims are assumed.

If the material interpretation of conditionals is correct, then "If S and T are thrown, then the motor starts" ($P \supset Q$) [1] is false only when the antecedent is true and the consequent is false. Grice points out that the conditionals in the purported counterexamples are consistent with this; they demonstrate rather that the conditionals should not be asserted. If I understand him correctly, he assumes that what should not be asserted cannot be formalized; if $P \supset Q$ cannot be formalized, neither can $\neg P |=_{NC} P \supset Q$.

Why should these conditionals not be asserted? The falsity of the antecedent or the truth of the consequent occurs in these scenarios. If the sentence "S and T are not thrown simultaneously" ($\neg P$) conveys as much information as $P \supset Q$, then it meets the maxim of quantity when $P \supset Q$ does not. $P \supset Q$ asserts more than is necessary. The same holds for the sentence "the motor is starting" (Q) in place of $\supset P$.

To refute the counterexamples, Grice relies on a suggestion which follows from conversational maxims; Jackson relies on a conventional suggestion about all propositions of the form $P \supset Q$. Conditionals have a specific purpose in Jackson's account. If a speaker asserts "if A, then B", then she is demonstrating that she accepts the necessary truth of B given A (*modus ponens)*. However, such a demonstration cannot occur in the counterexamples.

Suppose a speaker believes the proposition "S and T are not being thrown" ($\neg P$), for example. If she is not informed about Q – "the motor is running" – then a statement of $\neg P$ is stronger than an assertion of $\neg P \vee Q$, which conveys more information than is necessary. It would nevertheless be appropriate to assert the latter as long as-and this is the crucial point-she believes $\neg P$. If P is found to be true, the disjunction would not be stated or would be withdrawn; she would not move on to infer Q by a negation of $\neg P$ in the disjunction and *modus tollendo ponens*.

Since $\neg P \vee Q$ is equivalent to $P \supset Q$, knowledge of P would also make $P \supset Q$ not highly assertible. The conditional "If S and T are being thrown, then the motor is starting" could not be operated on by *modus ponens*.

This distinguishes the conditionals in the counterexamples from those which are "robust" enough to be believed when their antecedents are true. They are not asserted merely because their antecedents are believed to be false, as in the above example. Take the principle that any proposition is either true or false (but not both): $T \vee F$[2]. This is highly assertible, according to Jackson, even when it is learned which disjunct is correct; $\neg T \supset F$ is highly assertible for any proposition and *modus ponens* can (a priori) operate on "If a proposition is not true, then it is false."

1 This is a valid formalization, though different from the one above, and serves the present purpose better.

2 Strictly, the formula should be written as $(T \vee F) \wedge \neg (T \wedge F)$; but the former conjunct in these cases is often written alone as a matter of convention.

Bibliography

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