Supervaluationism, Dynamic Supervaluationism, and Higher-Order Vagueness

Wiktor Przybrorwski, *University of St Andrews*

The fact that the phenomenon of vagueness can itself be vague—and its vagueness be vague as well—seems impossible to make sense of without getting a headache. This so–called higher–order vagueness makes theorising about vagueness a notoriously difficult task for philosophers of logic and language. This difficulty manifests itself in that, even if a theory can convincingly explain what vagueness is and how we can reason about it, when faced with the vagueness of the just–tamed vagueness, it gets flooded with paradoxes and makes the initial theory seem implausible. In this paper, I argue that Rosanna Keefe's supervaluationism is one such theory. Even though it elegantly accounts for the first order of vagueness, it becomes less elegant when questioned about the higher orders. To demonstrate this, I show that Keefe's system fails to resolve various paradoxes of higher-order vagueness such as the finite series paradox or the D^* paradox. Furthermore, I argue that in her attempts to accommodate the paradoxes by adopting a rigid hierarchy of metalanguages, Keefe invites new worries. Given these criticisms, it is unlikely that Keefe's theory can be 'argued out' of these paradoxes—'finite series' in particular. Instead, I argue that the theory must be substantially modified if it is to be salvaged, and one way to do so is by making the proposed structure more dynamic. I attempt to do so by sketching an outline of dynamic supervaluationism that can tackle the problems that Keefe's supervaluationism cannot. I close my essay by teasing out some challenges that the proposed theory could face and offering possible solutions. I believe that supervaluationism is a very attractive approach to vagueness and therefore, it is worth developing further into a more robust theory that could tackle its higher orders.

1. Introduction

Vagueness in language refers to an indeterminate relationship between its terms and the world they describe.[1] Minimally, a predicate is vague if it has three features: **admission of borderline cases** (objects to which its application is unclear), **a lack of known, sharp boundaries** (no clear case separating the positive and negative cases), and (apparent) **susceptibility to the Sorites paradox**.[2]

Vagueness is philosophically relevant because it raises two problems. First, the **semantic problem**: since the vague extension is unclear, classical semantics (where meaning is derived from extension), and hence classical logic, may not apply. Second, the **Soritical problem**. Consider a series of people of descending heights by 1cm. The first is clearly tall (200cm) and the last is clearly not (120cm). Since no known boundaries exist, vague predicates are tolerant—a small change will not alter the application. Thus, by inductive step, for any case **n**, 'if **n** is tall then **n** + 1 is tall'. Starting at 200cm is tall, via a series of conditionals, you validly conclude that 120cm is tall. However, this is a contradiction since 120cm is clearly not tall.[3] This argument exemplifies the classical form of the Sorites paradox.

Theorizing about vagueness involves accounting for the nature, source and meaning of vagueness, providing vague semantics and resolving the Sorites. Furthermore, since it is unknowable where the positive extension changes to negative, it is equally unknowable where the positive changes to borderline. Thus, borderline cases themselves should be unbounded; hence there should be borderlines to borderlines. The process could be iterated to establish a possibly infinite hierarchy of borderline cases: the higher–order

[1] Kit Fine, *Vagueness: A Global Approach* (Oxford Academic, 2020), 2-3.

[2] Rosanna Keefe, *Theories of Vagueness*, (Cambridge University Press, 2000), 6-7.

[3] Fine, *Vagueness*, 3-7.

vagueness (HOV).[4]

Throughout this paper, I will follow Rosanna Keefe and other major supervaluationists in assuming that HOV is a genuine problem, that needs to be accounted for. However, it is worth pointing out that this is a debated matter in the field.[5] Nevertheless, under this assumption a successful theory of vagueness, given its commitments, must also account for HOV.

In this essay, I explore how one theory of vagueness—supervaluationism, advocated by Rosanna Keefe—does so. First, I outline her account of first–order vagueness (FOV). Then, I explain the problems posed by HOV, examining Tim Williamson's criticisms of the theory and how Keefe accommodates them. I will argue that although the Williamson problems are solved, the resulting view does not reflect how language actually functions and is paradoxical, making the HOV account unsatisfactory. I then attempt to modify the view by dynamizing it, developing the ideas of Hao–Cheng Fu. I defend the model by showing how it solves some of the critical issues faced by Keefe. Lastly, I raise a few possible issues endemic to the dynamic view and sketch responses to defend it.

2. Supervaluationism, a theory of vagueness

Supervaluationists claim that vagueness is a problem of language, not our epistemic capacities. They argue that vague predicates fail to draw sharp boundaries, not that these boundaries are unknowable, and that they admit borderline cases. The source is semantic indecision. A vague predicate admits a range of possible extensions, but it is semantically unsettled which one is correct. This is captured through the notion of precisification, a way to make a vague term precise.[6] A precisification must be admissible, reasonable in not licensing a misuse of language.[7] It also must be complete, it categorizes objects into positive and negative extensions, leaving nothing in–between. For illustration, consider the vague predicate 'tall'. We could (reasonably) use precisifications: 'tall' is true if '>175cm', '>180cm' and '>190cm', each of which would precisely divide objects into positive and negative extensions. Vague terms do not 'choose' between these; instead, all precisifications are equally good.[8]

Supervaluationists provide semantics for vague predicates, identifying truth with super–truth by considering all possible precisifications. $F\mathbf{a}$ is super–true (-false) iff F is true (false) of **a** under all complete and admissible precisifications. $F\mathbf{a}$ is neither true nor false iff F is true of **a** under some precisifications and false of **a** under others.[9]

Thus, vague predicates divide objects in a three–fold manner, where borderline cases are not assigned a definite truth value. Hence, supervaluationists give up bivalence, departing from classical semantics, by admitting truth value gaps. On the other hand, classical logic is mostly preserved because if a sentence is classically true, then it is true on all complete and admissible precisifications. Consider the law of excluded middle. Using any precisification of tall—every object will be either tall or not-tall, since every precisification divides objects into two sharp sets. Similarly, all classical theorems are retained, thus we can use classical logic to reason about vague predicates.[10]

This idea provides a straightforward solution to the Sorites. Namely, the inductive premise 'if $F\mathbf{n}$ then $F(\mathbf{n}+1)$' is super–false, since the antecedent will be true and the consequent false for some **n** under any complete and admissible precisification. This is because each precisification, being complete, provides a sharp cut–off between the true and false—a bordering pair where the first entry is true and second one

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4 Keefe, *Theories of Vagueness*, 31-32.

5 Some philosophers, such as Dominic Hyde, claim that higher–order vagueness (HOV) is a pseudo–problem, arguing that the vagueness of vague is a real, but unproblematic, phenomenon. Others, including Hao–Cheng Fu and Susanne Bobzien counter that this stance fails to adequately address the complexity of the issue, maintaining that HOV is indeed a genuine problem. While an extensive discussion is beyond the scope of this essay, see Hyde, "Why Higher–Order Vagueness Is a Pseudo-Problem"; Fu, "Saving Supervaluationism from the Challenge of Higher-Order Vagueness Argument"; and Bobzien, "In Defense of True Higher-Order Vagueness" for further details.

6 Keefe, *Theories of Vagueness*, 154-156.

7 Timothy Williamson, *Vagueness*, (Routledge, 1994), 158.

8 Keefe, *Theories of Vagueness*, 154-156.

9 Keefe, *Theories of Vagueness*, 154.

10 Rosanna Keefe, "Vagueness: Supervaluationism," *Philosophy Compass* 3, no. 2 (2008): 162-164.

is false.[11] Thus, the supervaluationist account fulfils the initial demands of theorizing about vagueness. Consult the footnote[12] for further clarification.

3. Supervaluationism and higher-order vagueness

The above metalanguage (talk of truth conditions) expresses the vagueness of the object language by dividing cases into three sharply bounded sets (true, false, borderline). This can be captured by adding a 'definitely' D operator to the object language, which functions akin to modal necessity.

The FOV of F is expressed as:

(1) DFx for definite positive cases (true under all complete and admissible precisifications)

(2) $\sim DFx$ & $\sim D\sim Fx$ for borderline cases (true/false under some)

(3) $D\sim Fx$ for negative cases (false under all)

This division is problematic since all cases are sharply categorized, allowing no borderlines between the definite and borderline cases, leaving no scope for HOV. Supervaluationists argue that this can be resolved by allowing the concept of 'admissibility' itself to be vague, thus making the metalanguage vague.[13]

Hence, the second–order vagueness of F is captured in the meta–metalanguage by expressing vagueness of DF (the metalanguage). This yields the following five–fold classification:

(1) $DDFx$, i.e., definitely definitely positive cases

(2) $\sim DDFx \& \sim D\sim DFx$, i.e., borderline between positive and borderline

(3) $D\sim DFx \& D\sim D\sim Fx$, i.e., definitely borderline cases

(4) $\sim DD\sim Fx \& \sim D\sim D\sim Fx$, i.e., borderline between negative and borderline

(5) $DD\sim Fx$, i.e., definitely definitely negative cases

The general idea is that for level vagueness of F, we need to show that **n** categories are vague. Thus, we need borderlines between those, in effect, drawing $2^n + 1$ categories.[14]

3.1. Williamson's challenge

Williamson argues that for this formalization to work, the D operator should not obey these two schemas:

(1) The S5 principle: If $\sim DF$, then $D\sim DF$.

(2) The S4 principle: If DF, then DDF.

If (1) and (2) hold, then whether a category is definite or indefinite, it will also be definitely so at higher levels. The supervaluationist cannot accept this since each category must be vague, otherwise it would draw sharp boundaries. Thus, Williamson recommends adopting a weaker modal logic, like T, with relative admissibility and no transitivity so that both S4 and S5 principles fail.[15] See the appendix for a more formal explanation.

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[11] Keefe, "Vagueness: Supervaluationism," 315-316.

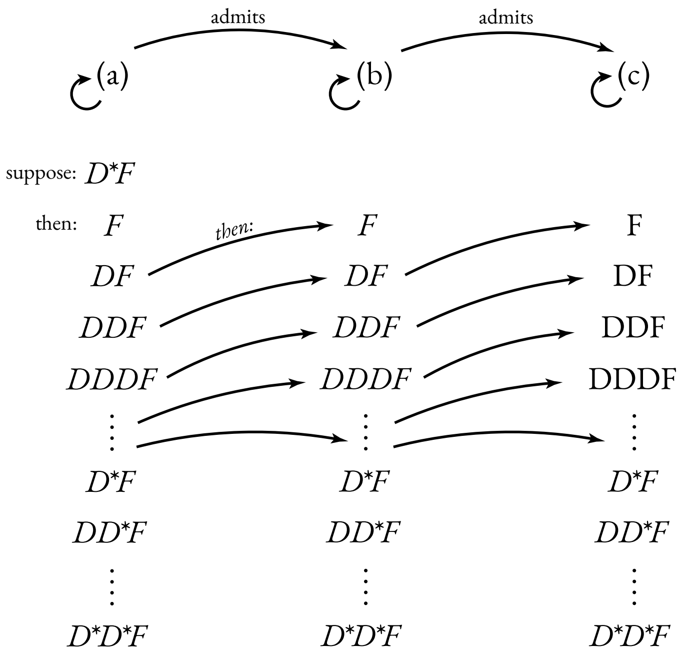
[12] Consider the series of people of varying heights again and suppose some examples of complete precisifications: **x** is short if (1) '< 160cm' or (2) '< 165cm' or (3) '< 170cm'. They are complete since they divide objects into positive (short) and negative (not–short) extensions with nothing in-between. It is easy to see how the inductive premise turns out false on each of these precisifications: (1) 'If 159cm is short, then 160 is short'; (2) 'If 164cm is short, then 165cm is short'; (3) 'If 169cm is short, then 170cm is short'. In each case, the antecedent is true and the consequent false (relative to precisification). Since the inductive premise turns out false for some pair under each complete precisification, it is super-false.

[13] Keefe, *Theories of Vagueness*, 202-204.

[14] Mark Sainsbury, "Concepts without Boundaries," in *Departing From Frege* (Routledge, 1990), 74.

[15] Williamson, *Vagueness*, 156-159.

However, Williamson argues that this is not sufficient to solve the problem via the D^* argument. He defines D^*F as an infinite conjunction F & DF & DDF &… & D_nF. Suppose precisifications (a), (b), and (c), where (a) admits (b), and (b) admits (c), but (a) does not admit (c), since admissibility is non-transitive. Suppose D^*F at (a). This means that $F, DF, DDF, \ldots, D_nF$ are true at (a). If DF is true at (a), then F is true at (b); if DDF is true at (a), then DF is true at (b); and so on. Thus, $F, DF, DDF, \ldots, D_nF$ are all true at (b), and hence D^*F is true at (b). The same reasoning applies to (c). Thus, if D^*F is true at some precisification, then D^*F is true at all precisifications. Hence, DD^*F is true at all precisifications—and by the same reasoning, so is D^*D^*F. Therefore, the S4 principle effectively applies to D^* (see diagram below).



If D^*F then DD^*F
If D^*F then D^*D^*F } S4 principle

Consequently, Williamson concludes that higher–order vagueness disappears.[16] This is because, for supervaluationism to succeed, each metalanguage must be vague. Thus, supervaluationists need a borderline case between D^*F and $D^*{\sim}F$, namely ${\sim}DD^*F$ & ${\sim}D{\sim}D^*F$. However, ${\sim}DD^*F$ collapses to ${\sim}D^*F$ by modus tollens on the S4 principle. ${\sim}D^*F$ then collapses to $D{\sim}D^*F$, given closure of D.[17] In effect, ${\sim}DD^*F$ & ${\sim}D{\sim}D^*F$ reduces to $D{\sim}D^*F$ & ${\sim}D{\sim}D^*F$ which is a contradiction. Since there are no borderlines to D^*F, it is not vague.

Williamson offers supervaluationists a way out: to give up semantic closure. D^* can be vague but its vagueness cannot be expressed using D or D^*. Instead, we need a meta–language for D^*, enriched with a distinct operator, *D*!. Then, to express vagueness of *D*!, we need a meta–metalanguage with *D*!!. Williamson remarks that the process could continue infinitely.[18]

Keefe takes up this proposal and advocates adopting an infinite, hierarchical series of metalanguages. In this model, the vagueness of the n^{th}-level metalanguage can only be expressed in the $(n+1)^{\text{th}}$ metalanguage,

[16] Williamson, *Vagueness*, 160.
[17] Patrick Greenough, "Higher-Order Vagueness," *Proceedings of the Aristotelian Society, Supplementary Volumes* 79 (2005): 183.
[18] Williamson, *Vagueness*, 160-161.

which is essentially richer than the nth language. She argues that, since there is no reason not to adopt such an infinite sequence, she can just stipulate that all the languages in the series are vague.[19] Greenough sketches a formalization where the object language is enriched with indexed D operators where each D_{n+1} is used to express the vagueness of D_n. Such formalization stops the D^* paradox and ensures that a non-vague metalanguage cannot be generated.[20]

4. Evaluation

Even though the above account might seem abstract, its strength lies in its simplicity—Keefe only iterates her account of the first order to higher orders of vagueness. In effect, the initial solutions to vagueness problems equally apply to HOV. Vagueness at higher orders remains a matter of semantic indecision: we are undecided over whether a precisification counts as admissible. Furthermore, each level n admits borderline cases and lacks sharp boundaries—a fact that can be expressed in the $n + 1$ metalanguage using appropriate D operators.

Moreover, each higher order metalanguage is still Sorites susceptible. I will explain this by running the paradox for the metalanguage (second order vagueness) in natural language terms for clarity—though the same could be done using D operators. The inductive premise for the metalanguage can be restated, in natural language, as: 'if there are admissible precisifications that draw the boundary to 'tall' at height h, then there are ones that draw it at one-hundredth of an inch lower'.[21] The second order series could start with a clearly admissible precisification (e.g., taller than 190cm) and end with a clearly inadmissible one (e.g., taller than 110cm). Since one–hundredth of an inch does not make a difference in admissibility, you could run a series of conditionals, starting with 'taller than 190cm is admissible' to reach a conclusion that 'taller than 110cm is admissible'. This is a contradiction. To resolve the second-order paradox, Keefe reuses her earlier strategy: for any complete way of making 'admissible' precise (or making 'definitely' definite), there will be a pair such that the first precisification is admissible and the second is not. This could be run for any level of metalanguage.

Thus, Keefe's account of HOV fulfils all the demands of a theory of vagueness. Each metalanguage is vague since it (1) admits borderline cases, (2) draws no sharp boundaries and (3) is Sorites susceptible. The fact that she achieves this for each order while maintaining her initial commitments (using the same technique at each order, characterising all levels of vagueness as semantic indecision, and so on) makes her strategy simple and elegant.

Even though this iteration neatly maintains the supervaluationist method, iterating to infinity is problematic. Keefe boldly claims that 'if there is no general objection to the claim that the sequence of metalanguages for metalanguages is infinite, then what is the difficulty with adding 'and each of those languages is vague' '.[22] However, there is a fundamental difficulty in this addition. In Keefe's system, the vagueness of an n-level metalanguage can only be expressed via an $n + 1$ level metalanguage. If all metalanguages are vague, then the infinite metalanguage would have to be vague. To express the vagueness of the infinite metalanguage, we would need to use the infinity +1 metalanguage. However, adding another element to an infinite set would not alter the size of this set.[23] Thus, the infinite +1 metalanguage would be on the same meta -level as the infinite metalanguage. Hence, the vagueness of the infinite metalanguage cannot be expressed and the statement 'each of those languages is vague' seems meaningless.

This objection points towards a more general issue with such Tarskian metalanguage hierarchies. Namely, that languages in such hierarchies cannot be globally quantified over.[24] Keefe could respond that even though the infinite metalanguage might not be definable in her structure, it does not mean that it does not exist. Her structure ensures that vagueness for any finite level can be expressed. Even though we cannot say that 'all metalanguages are vague', we also cannot identify any non–vague metalanguage within the structure.

[19] Keefe, *Theories of Vagueness*, 202-208.

[20] Greenough, "Higher-Order Vagueness," 184-186.

[21] Keefe, *Theories of Vagueness*, 207-208.

[22] Keefe, *Theories of Vagueness*, 208.

[23] MIT OpenCourseWare, *Session 11: Mathematics for Computer Science, 6.042J: Mathematics for Computer Science, Spring 2015* (Massachusetts Institute of Technology, 2015).

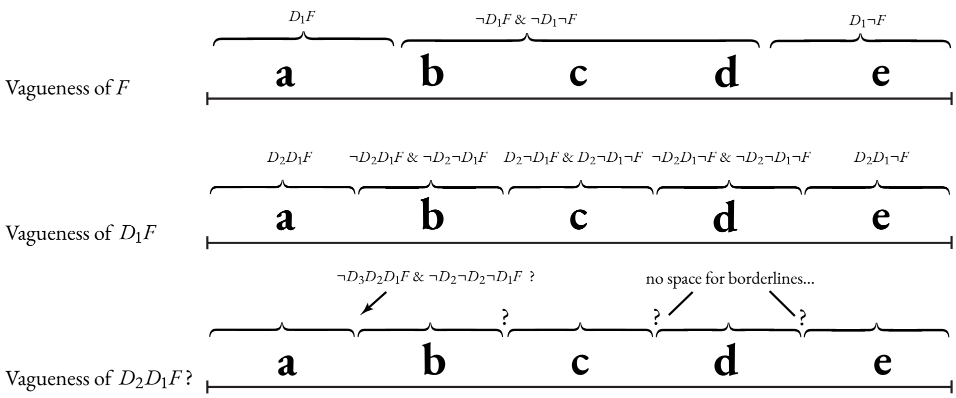
[24] Greenough, "Higher-Order Vagueness," 187.

Thus, even though the concept of infinity proves problematic for Keefe at the outset, I will assume that this problem does not threaten the explanatory power of her structure.

A further problem with the structure is that it is highly detached from how language functions. Competent speakers would find making sense of iterated uses of 'definitely' difficult, whether it is indexed or not. For example, saying someone is 'definitely definitely definitely tall' has little meaning apart from emphasis. Keefe might respond by pointing out that we do not use expressions like 'a googol of a googol of a googol' in ordinary conversation either, yet this does not mean the concept of 'googol' is not a meaningful mathematical concept. However, the issue goes deeper. As Saul Kripke pointed out, we cannot consistently assign levels to truth. Thus, even if we index the levels of 'definitely', it is difficult to assign them consistently. Consider the following statements: Jan says, 'Everything Alfred said is definitely false', and Saul says, 'Everything Jan said is definitely false'. To make sense of these, we would need to place one at a higher level in the hierarchy. However, this does not happen in natural language.[25]

Keefe might counter these natural language intuitions by arguing that her model is only an idealization which is not meant to exactly replicate how ordinary language works. While iterating 'definitely' (e.g., $D_3 D_2 D_1 F$) may make little sense in casual conversation, the model is primarily defended by its explanatory power regarding HOV. She could further argue that even though different levels of metalanguages, when expressed in natural language, might not be clearly marked and distinguishable (such as in the Jan–Alfred example above), they can still function as distinct metalanguages in a formal framework. A further worry is that such an approach might over–idealise HOV making her account arbitrary. It raises the question over whether speakers genuinely use implicitly distinct levels of metalanguages to assign levels to truth. Thus, Keefe would need to give a more robust explanation of the relationship between her model and natural language.[26]

Lastly, even though Keefe's iteration method allows her to respond to Williamson's D^* paradox and establish that there cannot be a non-vague metalanguage, the non-vagueness of each metalanguage requires further borderline cases. We need $2^n + 1$ categories to express the vagueness of the nth metalanguage. However, there is a tension between an infinite number of categories and a finite number of objects in the series: the finite series paradox. Consider a simple series with 5 objects. To account for 1st level, we divide them into 3 categories. To account for 2nd level, we divide them into 5 categories. At 3rd level there are 9 categories to be filled but only 5 objects. This means that at some level we will run out of objects with which to fill the categories. As a result, there will be no borderline cases between the categories - providing a sharp boundary, as pictured below.[27] Whether or not Keefe indexes her D operators makes no difference, there will always be an insufficient number of objects in the series to fill all categories.



In conclusion, even though the rigid hierarchy in Keefe's structure might be defended to some extent, her

[25] Saul Kripke, "Outline of a Theory of Truth," *The Journal of Philosophy* 72, no. 19 (1975): 694-697.

[26] A full discussion of this issue is beyond the scope of this essay, though the problem would require further explanation to defend the account effectively.

[27] Greenough, "Higher-Order Vagueness," 180; 185-186.

appeal to an infinite hierarchy is fundamentally in conflict with the finite Sorites. There seems to be no way to accommodate the problem without making strong alterations to the model.

5. 5. Positive proposal — dynamizing supervaluationism

5.1. 5.1. Introducing dynamic supervaluationism

I believe that Keefe's problems can be addressed by making the structure's categories dynamic. My proposal is loosely based on Hao–Cheng Fu's model.[28] Fu rejects Keefe's claim that admissibility is vague and instead claims that, when considering a vague predicate, we are using a well–defined set of precisifications (p–sets). Keefe might argue this counterintuitive since we do not know what is admissible. However, this knowledge is unnecessary: the p–set is created when cases are categorized as true, false, or borderline at time t_1. For example, if 195cm and 190cm are tall, 170cm is not, and 180cm is borderline, the p–set is implicitly formed dividing cases into three groups, on my reading of Fu. Crucially, we judge first; the p–set is constructed afterward. What follows in the next paragraphs is my own development of the idea.

Fu applies the AGM theory[29] to give a complex account of the dynamics of p–sets; however, offers little formalisation and does not explain how this idea could be applied to the challenges of HOV[30]. Moreover, Fu does not address the paradoxes of HOV, and it is difficult to see how his account could solve them. In my view, we do not need such an elaborate account. I propose that a p–set is dynamic solely in virtue of changing when a case is judged inconsistently with it. For the sake of clarity, consider the above example again. Imagine another person, **x**, who is 168cm. You judge **x** as tall. This is clearly inconsistent with your p–set at t_1, since you judged 170cm as not tall. Thus, adding **x** to the tall category updates the t_1 set to the t_2 set with revised precisifications. This change occurs by either (1) expanding (adding a precisification), (2) contracting (removing one), or (3) both. Therefore, I retain the core idea of dynamic p–sets and Fu's terminology but limit the scope of the mechanism to a minimal principle: a p–set updates only when a judgment is made that conflicts with it.

I will now attempt to formalise the above proposed working of p–sets, which I will later apply to the challenges haunting supervaluationism. Vagueness, on the dynamic view, remains semantic indecision. At the first level, we follow Keefe's supervaluationism with a slight addition of the temporal component. While Fu does not offer a formalisation of his view in the spirit of Keefe's system with D operators, the following temporal framework develops my own way of modelling dynamic p–sets using temporally indexed D operators.

More precisely, at any time, t, cases divide into D_tF, $D_t{\sim}F$, and ${\sim}D_tF$ &${\sim}D_t{\sim}F$: that is true, false, and borderline. However, unlike in Keefe's view, HOV arises not from undecided admissibility of a precisification but from the instability of precisifications. Suppose that you make some categorisations at t_1. According to the p-set that you just formed; some arbitrary case is classified as D_1F. Now suppose that you consider the series again, but you are no longer sure about the definiteness of your classification. Thus, your p–set is adjusted at t_2, and according to it, the case is borderline. Therefore, from t_2's perspective it was a borderline definite case at t_1 (${\sim}D_2D_1F$).

In general, when considering a borderline case after categorisation at t, tolerance ensures a mis–categorisation. To see this, remember that the supervaluation technique divides cases sharply into true, false, and borderline. However, tolerance guarantees that when viewing two neighbouring cases, we will not be able to tell the difference. Therefore, there is a clear tension; we divided sharply, enabling a border pair where, for instance, one member is true and another borderline. However, since we cannot distinguish between neighbouring cases, they must be categorised equally. That means that one of the cases had to be categorised mistakenly

[28] Hao-Cheng Fu, "Saving Supervaluationism from the Challenge of Higher–Order Vagueness Argument," in *Philosophical Logic: Current Trends in Asia* (2017), 147-152.

[29] AGM refers to the Alchourrón–Gärdenfors–Makinson model of belief revision, which accounts for rational change in epistemic states represented as belief sets. The theory outlines how agents should expand, contract, or revise their beliefs while preserving logical coherence. For more detail, see Carlos E. Alchourrón, Peter Gärdenfors, and David Makinson, "On the Logic of Theory Change: Partial Meet Contraction and Revision Functions," *The Journal of Symbolic Logic* 50, no. 2 (1985): 510–30.

[30] Fu, "Saving Supervaluationism from the Challenge of Higher-Order Vagueness Argument," 149-152.

and thus, the p–set must be revised to maintain consistency in our judgments. When we reconsider the series at t_2, the earlier categorisations from t_1 turn out to be indefinite, as case memberships shift.

5.2. Applying dynamic supervaluationism

Having formalised the view, I will now apply it to the challenges of HOV, starting with Williamson's D^* argument. To attack the dynamic approach, D^* could be restated as the conjunction 'DA at t_1 & DA at t_2 & DA at t_3 & … & DA at t_n'. As discussed in section 3, the D^* argument establishes that, if D^* is not shown to be vague, then the cases where D^* is true and the cases where D^* is false will both be ultimately definite. Hence, there will be no borderline cases between D^* categories, which provides a sharp boundary. This contradicts the foundational supervaluationist claim that there are no sharp boundaries. However, this argument loses its force under the dynamic view. The dynamic framework allows us to easily account for the vagueness of D^*. Just as in the case of any D, we need to progress in time to express D^*'s vagueness. Thus, while D^* may initially appear to be non–vague, this is because we need to move to $t + 1$ to realize its vagueness.

Secondly, Keefe's view faced concerns about rigid hierarchies, but the dynamic approach eliminates these. When two speakers disagree over a case's definiteness, neither statement must be 'prior'. They are simply speaking from different p–sets that underwent different evolutions. There is no rigid hierarchy of metalanguages since each discusses categorizations in another metalanguage, and no pair can be clearly ranked as 'prior'.

This lack of priority arises because it would be impossible to assign it to any particular metalanguage. Surely, the metalanguage at $t + 1$ must be a metalanguage of the metalanguage at t, since it is able to express facts about t. Therefore, it is more 'privileged' in this sense. However, suppose that the p–sets evolve over time such that, when moving from $t + 1$ to $t + 2$, we go back to the original p–set from t. Then, the t and $t + 2$ metalanguages gain their truth conditions from the same p–set. Therefore, in a sense, the t metalanguage becomes 'prior' to the $t + 1$ metalanguage. This would undermine the strict, unidirectional Tarskian hierarchy.

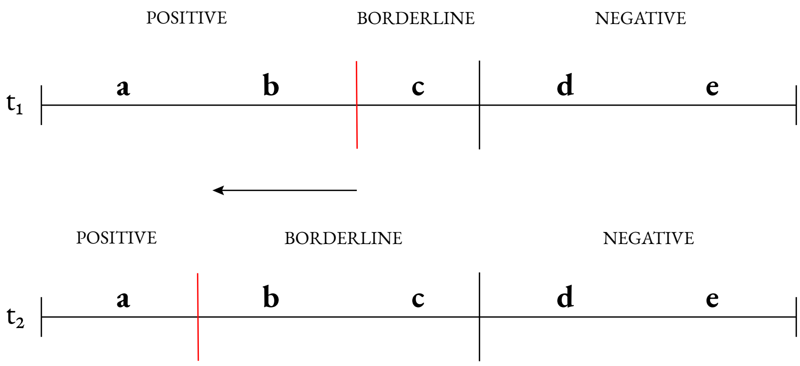
One could further argue that we could suppose a scenario in which two identical people, A and B, undergo identical p–set evolutions. However, A's evolution stops at t and B's evolution stops at t+1. On the one hand, we might be tempted to assign priority to B's statements, which would be counter–intuitive on the natural language objection. However, there is no reason to suppose that A's evolution should go the same way; she might consider a different part of the Sorites spectrum. Therefore, although the metalanguages are in some sense hierarchical, none has a clear priority in determining the truth of one classification over another. Thus, the objections, such as the ones made by Kripke, do not apply here.

Thirdly, the dynamic view can help tackle the finite series paradox, which was a critical blow to Keefe's account. I will explain how it could achieve this through an example. Consider a 5–element Sorites with objects **a**, **b**, **c**, **d**, and **e**. Suppose that Alfred's initial categorizations are:

$$\begin{aligned} D_1F &= \{\mathbf{a}, \mathbf{b}\} \\ \sim D_1F \,\&\, \sim D_1{\sim}F &= \{\mathbf{c}\} \\ D_1{\sim}F &= \{\mathbf{d}, \mathbf{e}\} \end{aligned}$$

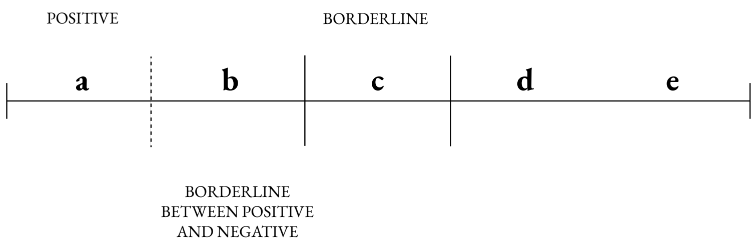
Alfred considers the pair **b** and **c** again. He realizes that he cannot tell the difference, concluding that **b** is also borderline. He adjusts his p–set accordingly, forming a new t_2 p-set.

$$\begin{aligned} D_2F &= \{\mathbf{a}\} \\ \sim D_2F \,\&\, \sim D_2{\sim}F &= \{\mathbf{b}, \mathbf{c}\} \\ D_2{\sim}F &= \{\mathbf{d}, \mathbf{e}\} \end{aligned}$$



The t_1 division, from the perspective of t_2 becomes:

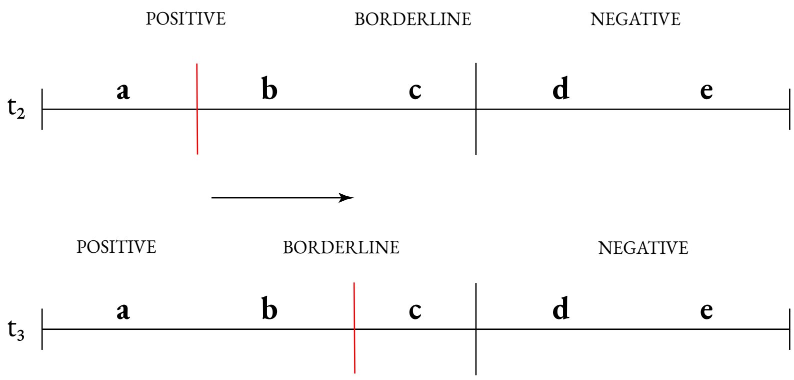
$$\begin{aligned} D_2 D_1 F &= \{\mathbf{a}\} \\ \sim D_2 D_1 F \ \& \sim D_2 \sim D_1 F &= \{\mathbf{b}\} \\ D_2 \sim D_1 F \ \& \ D_2 \sim D_1 \sim F &= \{\mathbf{c}\} \end{aligned}$$



Hence, in this part of the series, the vagueness of D_1 is fully accounted for since all D_1 categories have borderline cases.

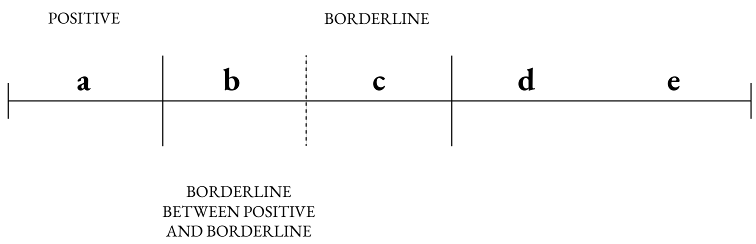
Now suppose that at time t_3, he looks at the pair **a** and **b**. Since he cannot tell the difference, he decides that b is also a definite case, adjusting the p–set again.

$$\begin{aligned} D_3 F &= \{\mathbf{a}, \mathbf{b}\} \\ \sim D_3 F \ \& \sim D_3 \sim F &= \{\mathbf{c}\} \\ D_3 \sim F &= \{\mathbf{d}, \mathbf{e}\} \end{aligned}$$



Since **b** changed its category membership, from the perspective of t_3, **b** was not a definite borderline case at t_2. Thus, the t_2 division, from the t_3 perspective, is:

$$\begin{aligned} D_3D_2F &= \{\mathbf{a}\} \\ {\sim}D_3D_3F \,\&\, {\sim}D_2{\sim}D_2F &= \{\mathbf{b}\} \\ D_3{\sim}D_2F \,\&\, D_3{\sim}D_2{\sim}F &= \{\mathbf{c}\} \end{aligned}$$



Thus, vagueness of D_2 is accounted for.

In general, any bordering pair will exhibit change when reassessed. Thus, any categorization at t can prove indefinite at $t + 1$. In effect, you will never reach a point where there are more categories than members in the series since the fluid categories will always be filled. An object can fill different categories at different times. This also does not mean that the t_1 categories are definite at t_3, only that their vagueness cannot be expressed from the t_3 perspective.

6. Addressing possible objections

Dynamizing supervaluationism provides new methods to tackle the paradoxes of HOV and other problems, for which standard supervaluationism struggles to account. However, it also presents new worries, which I will explore and sketch responses to in this section of the essay.

6.1. Fixed time worry

The first possible objection to the view is that it breaks down when time is fixed. This is because the account of HOV relies on shifty p-sets, which in turn rely on the progress in time. More precisely, the vagueness of some set of categories drawn in period t can only be expressed in period $t + 1$. Thus, if we hold the time fixed, the view breaks down: the categories drawn in period t appear to be sharply bounded, which contradicts the foundational claim that there are no sharp boundaries.

Although this might seem like a critical blow to the view, there are two possible lines of response. First, we could simply reject the inference from our inability to express the vagueness of some order when time is fixed, to the claim that there are sharp boundaries. After all, the fact that we cannot express it does not imply that it does not exist. This, however, demands further explanation of why we cannot express it. One response is that at a certain time, we are just using a well–defined but arbitrary set of precisifications. However, this division is surely wrong; it is made under one of many sets of equally good precisifications. Thus, there is no reason to believe that the term was made precise—we just have not realized our mistake yet.

A second and more powerful response is to deny the possibility of fixing time in this sense. This could supplement the above argument. Suppose that the critic of the view wants to prove to us that there are sharp boundaries. However, in order to show that there are sharp boundaries, they would have to find them in the series. Suppose that you manage to find the extension–switching pair. Even if you do this, you will realize, per tolerance, that you cannot tell the difference between the two cases. In effect, you must conclude that one of the cases was falsely classified when you made the division in the previous period. Thus, your p–set changes. Therefore, the very considering of the sharp distinction would automatically progress us to $t + 1$, ensuring that there was no sharp boundary. In conclusion, the fixed time objection is not a significant worry to the dynamic view.

6.2. Collapse to contextualism worry

There is a second and more dangerous worry: one could argue that the supervaluationist aspect of the dynamic view seems unimportant. By this, I mean the use of supervaluationist semantics and classification of vagueness through indecision between precisifications. It is only directly applied to resolve FOV, and one could argue that the relativity of classifications over time, which accounts for HOV, could be equally applied to FOV. In effect, the supervaluationist method would disappear. If this argument is accepted, and if we further assume that the functioning of p-sets is sufficiently similar to that of contexts, then the dynamic view risks collapsing into a contextualist one. This could have some benefits, such as the preservation of bivalence (which contextualists keep) and making the view more parsimonious by unifying the approaches to vagueness at different orders.

In what follows, I will defend the dynamic view from this objection. See footnotes for background on contextualism[31] and their solution to the Sorites.[32] The first point that I address is the idea that supervaluation is obsolete. On this view, its role at the first level could be replaced by the context–reminiscent p–sets. The intuitive idea is that, since shifty p–sets account for HOV, why not apply them to FOV and get rid of additional semantic claims and concessions altogether? However, this intuition is misguided, since the supervaluationist solution to FOV is required to make the shifty p–set account of HOV work. This is because the first–order divisions allow for the p–sets to shift in the first place. At the first stage, we implicitly categorize objects into positive, negative, and borderline cases. These categories are directly determined by the p–set, which sets out the supervaluationist truth conditions (i.e., *DF* iff true for all precisifications and so on). These categorizations are provisional: they impose sharp boundaries where none truly exist. This tension allows for future revisions of p–sets, and thus for p–sets to shift. Hence, without supervaluation in the beginning, the p–sets cannot shift. And if they cannot shift, they cannot account for any order of vagueness.

A stronger claim could be made that the p-sets are entirely purposeless if we do not allow for supervaluation. To see the point, imagine that you have some set of precisifications of tall {> 170cm, > 180cm, > 190cm} and you use them to categorize a group of people in the series. Without supervaluation, you end up with six extensions, i.e., three positive and three negative extensions, one per precisification. There are no borderline cases, since without supervaluationist truth conditions—where borderlines are true under some precisifications and false under others—such cases are not defined. Since this is a key symptom of vagueness, as stressed in the beginning, this result would require further explanation of why we think there are borderlines at all.

An enemy of the view could argue that this response misses the point—vagueness did not fail to arise

[31] Contextualism rests on the claim that vagueness is a species of context-sensitivity. This roughly means that, in its application across different contextual circumstances, a vague term maintains a constant *character* but shifts in *content*. Therefore, vague terms function like indexical terms. The relationship of vagueness and indexicality is a contested matter for contextualists. Some hold that vague terms behave *like* indexicals, while others claim they *are* indexicals. However, this distinction is not directly relevant to the discussion, and the objections raised here apply equally to both views. Consider the word *now*. It adheres to the same grammatical rules (i.e., has the same *character*) when uttered today and tomorrow. However, when said today, it picks out a different time than it does when used tomorrow (i.e., has different *content*). Similarly, a vague predicate like *tall* is used in the same way when applied to members of a group of pygmy peoples, as when applied to a group of Dutch people. However, it would pick out radically different people. In the first case, the extension of *tall* likely includes some of the world's shortest people; in the second, some of the tallest. See Roy Sorensen, "Vagueness," *The Stanford Encyclopedia of Philosophy* (Winter 2023 Edition), ed. Edward N. Zalta and Uri Nodelman.

[32] Contextualists exploit this idea of unstable extensions over contexts to solve the Sorites by accusing it of equivocating different meanings of a vague term. Similarly to the supervaluationists, the contextualists target the inductive premise (2). The contextualist is committed to the claim of weak tolerance (WT), which states that when two members of a bordering pair are considered in the same context C, they will belong to the same extension. However, WT permits that when one member is considered in context C and the other in C', then they might belong to a different extension. See Jonas Åkerman and Patrick Greenough, "Hold the Context Fixed—Vagueness Still Remains," in *Relative Truth*, ed. Manuel García-Carpintero and Max Kölbel (Oxford University Press, 2010), 275–76.

The WT explains why the inductive premise seems to hold. If we consider any pair in the series, we will conclude that both members belong to the same extension. But this is just because we are disposed to view them in the same context C. The contextualist says that, in fact, the context will gradually change across the series. This means that even if we classify neighbouring terms the same at first, this classification will not persist throughout the series. Thus, the inductive premise of the sorites, such as 'if **n** is short, then **n** + 1 is short', fails since the meaning of 'short' is not the same for every member **n**. This is because, the shift of context C into C', enables cases where '**n** is short' is true (in C) but '**n** + 1 is short' is false (in C'). See J. Åkerman, "Contextualist Theories of Vagueness," *Philosophy Compass* 7 (2012): 470–75.

due to the absence of supervaluation, but rather because the p–sets did not shift. After all, on the dynamic account, it is the shiftiness of p–sets that allows for HOV. To address this, let us suppose, for the sake of the argument, that the p-set can somehow shift without supervaluation. Imagine, for instance, that the p–set expands by incorporating an additional precisification to the set. You now have eight extensions, yet still no explanation for either first–order or higher-order vagueness. Thus, even with shifty p–sets, the dynamic view cannot function without supervaluation, showing it to be an essential, not merely supportive, component of the account.

Therefore, the case for the contextual collapse breaks down in the very beginning. We simply cannot make the p-sets shifty without maintaining the baseline supervaluationist aspects of the theory. If we cannot make the p-sets shifty, they cannot resolve FOV, let alone HOV. Hence, supervaluation is by no means obsolete. However, to strengthen the defense, I will demonstrate that the next step needed for the contextualist collapse fails. That is, I will show that p–sets and contexts behave very differently.

Although they might appear similar, the former crucially relies on the characterization of vagueness as semantic indecision, while the latter depend on context sensitivity. We might express this difference by saying that the p–sets are inward–oriented, while contexts are more outward–oriented. This is because the former shifts due to our indecision among several equally good precisifications at the initial stage. This indecision prompts us to make mistakes, which we subsequently correct by revising the p–set into another equally acceptable p-set. Thus, the changes directly follow our judgments. By contrast, shifts in contexts seem to have an effect on our judgments—contexts shift first, and judgments follow. Thus, the machinery appears to be quite different.

One could even argue that shifty p-sets rest on a firmer theoretical ground—their shiftiness is caused by our inconsistent judgments. On the other hand, the contexts appear to shift arbitrarily. Thus, the contextualist requires some external justification for this instability. Additionally, the contextualist needs to show how contexts could become shifty enough to prevent every instance of the Sorites. In other words, enough shiftiness must be generated. I do not intend to digress further, but the key takeaway is that despite their apparent similarities, p–sets and contexts differ significantly. Thus, the threat of the 'collapse' does not seem to be so imminent.

As a final point to strengthen my argument, I will provisionally assume that the dynamic approach could collapse into contextualism. Even in such a scenario, there remain independent reasons to prefer the former view over the latter. One significant reason is that contextualism undermines some of our most basic approaches to reasoning. Contextualism requires extensions of vague terms to be unstable, which is precisely what enables it to defeat the Sorites. However, these shifty contexts become deeply problematic when applied outside of the paradoxical setting.

To see this, consider the following example. Saul and Jan are borderline cases of tall. The former is 176.1cm, and the latter is 176cm. Suppose you judge both of them to be tall. Now consider applying the following instance of conjunction introduction:

$$\frac{\text{Saul is tall} \qquad \text{Jan is tall}}{\text{Saul and Jan are tall}} \wedge \text{I}$$

However, if the extension of the vague predicate *tall* is unstable, we can easily imagine a situation in which both premises are individually true, yet the conclusion turns out false. This would happen if the context shifted midway through the argument. Thus, although context sensitivity is useful for solving the Sorites, it is dangerous when applied to everyday reasoning. Specifically, how can contexts remain sufficiently stable to ensure our logic does not fail even in such simple cases?[33]

In contrast, dynamic supervaluationism does not provoke such worries. Under supervaluationism, the rule of a conjunction introduction always preserves validity. To illustrate, consider a p–set representing precisifications for *tall*: {>170, >175, >176}. First two precisifications make both premises true and the conclusion true as well. The third precisification makes one of the premises true, the other false, and the conclusion false. This will work for any possible precisification. Consequently, it applies to every p-set.[34]

[33] J. Åkerman, "Contextualist Theories of Vagueness," *Philosophy Compass* 7 (2012): 475–76, .

[34] This follows the exact same reasoning as that applied to the failure of the inductive premise or the truth of the law of excluded middle discussed in more detail at the beginning of the essay.

One might argue that, similarly to a shifting context, the p–set could shift over the course of an argument. For example, we might initially classify both premises as true (e.g., using the set {>170, >175}, but later we classify the conclusion as false (e.g., shifting to the set {>177, >180}). However, this objection reflects a misunderstanding of supervaluationist semantics, since arguments must always be evaluated relative to a single p–set. If we shifted the p–set to the second one, both premises would become false along with the conclusion. Therefore, the validity of conjunction introduction would remain intact.

Why is this strategy not available to the contextualist? The contextualist could simply deny that contexts can shift in such ways, insisting instead that we always evaluate the premises and the conclusion within a single context. However, this directly contradicts the contextualist's equivocation strategy to the Sorites paradox. That is, the strategy according to which bordering cases may differ in truth value because their evaluation contexts differ. Hence the contextualists need contexts to shift. In effect, they cannot deny that the above scenario is possible. Instead, their strongest response would likely be to argue that such cases rarely happen.

I do not intend to argue that supervaluationism, or its dynamic version, is superior to contextualism. Such a claim is clearly beyond the scope of this essay and perhaps beyond the scope of any single essay. Rather, my point is simply that there are independent reasons to prefer the dynamic view over contextualism. Therefore, the claim that contextualism explains everything that the dynamic view explains—but more simply, and thus more parsimoniously—is clearly not accurate.

Taking stock of these considerations, the collapse argument fails not only at its initial stage but also on all subsequent fronts. Dynamic supervaluationism is by no means contextualism in disguise; rather it is its own theory, deeply grounded in Keefe's original supervaluationist framework.

7. Conclusion

While Keefe's supervaluationism remains an attractive account of vagueness, it ultimately struggles to account for higher–order vagueness. Her adoption of a rigid, Tarskian infinite hierarchy may block Williamson's D^* argument, but at the cost of disconnecting the theory from natural language. Even if, as I briefly explored, she could respond to these problems, adopting an infinite metalanguage hierarchy still leaves Keefe subject to a seemingly unresolvable finite series paradox. I argued that Keefe's account could be dynamized by incorporating ideas from Fu, thereby resolving the finite series paradox and avoiding issues associated with a rigid hierarchy. Yet, the dynamic model itself introduces new difficulties, notably the 'fixed time' and 'collapse to contextualism' problems. To defend the view, I briefly outlined potential replies to these issues, showing that they are not fatal. Dynamizing supervaluationism may not resolve all problems, but it is a promising development of the supervaluationist theory and would be worth elaborating on and defending in future enquiries.

Appendix

Why must Keefe deny the S4 and S5 principles?

(1) The S5 principle: If $\sim DF$ then $D\sim DF$.

(2) The S4 principle: If DF then DDF.

Suppose that (1) and (2) hold and that we have the first–order classification:

(i) DF for definite positive cases.

(ii) $\sim DF$ & $\sim D\sim F$ for borderline cases.

(iii) $D\sim F$ for negative cases.

If (1) holds, it implies that at the second level, DF and $D\sim F$ transform into DDF and $DD\sim F$ (see proofs a and b). That is, the definite positive and definite negative case is definitely definite positive and definitely definite negative, subsequently. If (2) holds, it implies $\sim DF$ & $\sim D\sim F$ $\sim DF$ & $\sim D\sim F$ transforms into

$D{\sim}DF$ & $D{\sim}D{\sim}F$ (see proof c). That is, the borderline case is definitely a borderline case. However, second–order vagueness would require two more categories—the borderline between positive and borderline (${\sim}DDF$ & ${\sim}D{\sim}DF$) and the borderline between borderline and negative (${\sim}DD{\sim}F$ & ${\sim}D{\sim}D{\sim}F$). As a result, sharp boundaries are drawn between the three categories since there are no cases between them.

Proof a:

$$\frac{DF \qquad DF \rightarrow DDF}{DDF} \rightarrow E$$

Proof a:

$$\frac{D\neg F \qquad D\neg F \rightarrow DD\neg F}{DD\neg F} \rightarrow E$$

Proof c:

$$\dfrac{\dfrac{\neg DF \rightarrow D\neg DF \qquad \dfrac{\neg DF \neg D\neg F}{\neg DF}\wedge E}{D\neg DF}\rightarrow E \qquad \dfrac{\dfrac{\neg DF \wedge \neg D\neg F}{\neg D\neg F}\wedge E \qquad \neg D\neg F \rightarrow D\neg D\neg F}{D\neg D\neg F}\rightarrow E}{D\neg DF \wedge D\neg D\neg F}\wedge I$$

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