A Defence of the Interpretational Account of Validity

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Both the interpretational account and the representational account provide contrasting accounts of validity for natural-language arguments. While the interpretational account captures formal validity, unlike the representational account, it does not capture materially valid arguments. Therefore, materially valid arguments are viewed as counterexamples to the interpretational account. I motivate why we may want to defend the interpretational account over the representational account and then proceed to defend the interpretational account using the suppressed premise strategy. The first objection to the suppressed premise strategy is by Stephen Read, who argues that the supressed premise is redundant. My contribution is to demonstrate how his objection fails. I also discuss and defend the suppressed premise strategy against other objections, which concern the nature of the supressed premise and the problem of modus ponens.

Introduction

Validity, a key concept in logic, concerns whether an argument is truth-preserving. The interpretational account of validity defends the view that for an argument to be valid it must be formally valid. I turn first to the importance of logical form, its role in logic, generally, and validity, specifically. My discussion then moves to the interpretational account alongside its rival, the representational account. Both accounts face distinct issues. While I do not hold that the representational account is incoherent, I do hold that its formulation has weaknesses that are absent in the interpretational account, giving a motivation for preferring the latter rather than the former. Materially valid arguments, which are not formally valid, present counterexamples to the interpretational account. The remainder of the essay is devoted to showing how the suppressed premise strategy can defend the interpretational account against this main objection. The suppressed premise strategy will in turn be defended against pressing objections, primarily Stephen Read's objection that the suppressed premise is redundant. This objection to the supressed premise strategy aims to prove that there is a contradiction in adding a suppressed premise to an already materially valid argument, and my contribution is to show how this objection fails. I then go on to defend the suppressed premise strategy against a few other objections, including objections concerning the nature of the suppressed premise and the argument, and the problem of modus ponens. The result is a defence of the interpretational account of validity, using the suppressed premise strategy.

Understanding the Relation Between Logical Form and Validity

Logic is considered the science of deduction: it deals with arguments and their validity. In formal logical languages, like truth functional logic and first order logic, we can capture validity using the standard notion of logical consequence. A formal argument is valid if the conclusion is a logical consequence of the premises. As Owen Griffiths and Alexander Paseau put it, "A formal sentence ϕ is a logical consequence of a set of formal sentences γ just if every model of γ is a model of ϕ".[1] Thus, we can describe the formal notion of validity for a logical language, using a model-theoretic notion of logical consequence.

Once we have captured the notion of validity for logical languages, we can move on to understanding the concept of validity as applied to natural language, as the accounts of validity that will be discussed are

[1] Owen Griffiths and Alexander Paseau, *One True Logic* (Oxford: Oxford University Press, 2022), 8.

accounts of validity for natural language. To understand validity as applied to natural language, we must introduce the concept of logical form. Logical form is generally considered to be a property of a sentence of natural language. The logical form of a sentence is when, keeping the logical constants fixed, the non-logical expressions get replaced with variables of the appropriate sort. Thus, the logical form of a sentence can be expressed using a schema. Given this schematic representation of form, we can follow Alfred Tarski in the view that logic is topic neutral, because a schema abstracts from the content of the sentence, only retaining the form of the sentence. For example, take the following sentence:

(1) Pigeons wear vests and cats wear hats.

This sentence can be expressed using the following schema:

(2) $A \wedge B$.

This is because the logical expression in sentence (1) is "and" which can be formalised using the symbol "$\wedge$", and the non-logical expressions in the sentence are "pigeons wear vests" and "cats wear hats", and thus these expressions are replaced with variables.

One stipulation with this account of logical form, is that it requires us to have an understanding of what a logical constant is. Thus far formality has been captured by its topic neutrality, and since a demarcation of logical notions is crucial to form, it makes sense to construct this demarcation using this quality of topic neutrality. Here we can invoke Tarski's account of isomorphism invariance. Tarski defines logical notions using an analogy from geometry. Just as we may demarcate particular geometrical objects by their invariance under transformations, so too can we demarcate logical notions. Thus, "we call a notion 'logical' if it is invariant under all possible one-one transformations of the world onto itself".[2] To explain this further, we can consider an isomorphism to be a bijective function, so between two structures there is a one-one mapping, which preserves all the relevant relations. This isomorphism is the transformation that Tarski is speaking of. For a relation to be isomorphic invariant it must remain unchanged over this sort of transformation. A relation that is isomorphic invariant is thus indifferent to individual objects. The only notions that do this are logical notions, and this confirms neutrality. Thus, we can define a logical notion as being isomorphically invariant and non-logical notions as not being isomorphically invariant. This allows for the demarcation, which is necessary to define logical form.

This understanding of logical form can now aid us in capturing the notion of formal validity for natural language. It is common in the literature to equate an argument being formally valid with it being valid in virtue of its form.[3] However, using this as a definition for formal validity is unsatisfactory, for we still need to define being valid in virtue of form, which I find to be no more informative than formal validity. Therefore, I define formal validity to be the following: an argument is formally valid iff it has a form which has only valid instances. An example of a formally valid argument is:

(3) All men are mortal, Socrates is a man $\therefore$ Socrates is mortal.

The logical form of the argument can be captured using a schema, as described above. Given the use of quantifiers in (3), the schema of the argument is simply its first order formalisation (on the obvious formalisation key):

(4) $\forall x(Fx \rightarrow Gx),\ Fa \ \therefore\ Ga.$

There are no invalid arguments with this form, therefore all the instances of this form are valid, consequently the argument is formally valid. It is clear from this explanation that this definition of validity for natural languages coincides with the definition for formal languages, meaning that a natural-language argument is formally valid iff its formalisation is valid.

[2] Alfred Tarski, "What are Logical Notions?," *History and Philosophy of Logic 7,* (1986), 149.
[3] Mark Sainsbury, *Logical Forms: An Introduction to Philosophical Logic*, (Oxford: Blackwell, 2001): 37.

Two Accounts of Validity

We can now examine two model-theoretic accounts of validity for natural-language arguments. Generally, model-theoretic accounts of logical consequence are now viewed as more successful compared to other accounts of logical consequence, and the two accounts that are the focus of this essay are model-theoretic. As such the central thesis of both accounts understands logical consequence as concerning truth preservation across models.[4] The first account is the interpretational account of validity, which originates from Bolzano but was promulgated by Tarski.[5] This account holds that an argument is valid if there are no possible interpretations of the argument (except for a reserved class of logical interpretations) where the premises are true and the conclusion false. An interpretation of an argument is any argument that has the same logical form as the initial argument. The second account is the representational account of validity, which holds that an argument is valid if it is impossible for the premises to be true and the conclusion false.[6]

The interpretational account only accepts arguments that are formally valid. The account achieves this by examining different logical interpretations of the argument; if there is no interpretation that has true premises and a false conclusion then the argument is considered valid. On the other hand, the representational account allows for arguments that are materially valid, alongside those that are formally valid. Materially valid arguments are arguments in which the validity of the argument is in part due to the meaning of the non-logical terms involved. An example of a materially valid argument is:

(5) Jill is a paediatrician ∴ Jill is a doctor.

The representational account intends to capture a more "intuitive" notion of validity. Defenders of this account hold that materially valid arguments are contained within this intuitive notion of validity, and so an account of validity must capture material as well as formal validity. This belief is rooted in the idea that there is an analytic connection between certain words or phrases, and these connections make the argument valid, even though the argument is not formally valid.

The main objection to the interpretational account is that it is subject to counterexamples, which take the form of materially but not formally valid arguments. To establish the success of the interpretational account we must meet this objection. One example of a materially but not formally valid argument is (5) above, and another is:

(6) Adam is taller than Bill and Bill is taller than Cathy ∴ Adam is taller than Cathy.

Neither of these arguments is formally valid, since there are invalid arguments with the same form as (5) and (6). The interpretational account would not accept that they are valid arguments given there are interpretations of (5) and (6) for which the premises are true and the conclusion false. A formalisation of these arguments in first order logic reveals their logical form:

(7) $Fa \therefore Ga$

(8) $(Tab \wedge Tbc) \therefore Tac$

Another interpretation of each of these arguments demonstrates the point further:

(9) Pat is a postman ∴ Pat is a father.

(10) Alice is friends with Bonnie and Bonnie is friends with Carl ∴ Alice is friends with Carl.

[4] This contrasts with proof-theoretic accounts which hold that the nature of logical consequence involves there being a proof from the premises to the conclusion.

[5] Jc Beall, Greg Restall, and Gil Sagi, "Logical Consequence", *The Stanford Encyclopaedia of Philosophy* (Summer 2024 Edition); Stephen Read, "Formal and Material Consequences", *Journal of Philosophical Logic* 23, no. 3, (1994): 249.

[6] Read, "Formal and Material Consequences", 250.

These arguments are clearly invalid, yet they have the same logical form as (5) and (6), respectively. It is due to these alternative interpretations that (5) and (6) are not valid.

However, the arguments (5) and (6) would be accepted under the representational account due to this account's use of modality. The representational account identifies logical consequence with metaphysical consequence. The reference to "impossible" in the representational account is a modal notion, whereas the interpretational account does not include such modal notions. The reference to "no possible interpretations" in the interpretational account may be made actual using substitutional classes, and thus does not need to rely on an analysis of modality.[7] Yet, it is because of its use of modality that the representational account can attribute validity to (5) and (6), for there is no possible world where the premises of (5) and (6) are true and the conclusion false.

On the other hand, modality is an issue for the representational account, for it requires that we have an analysis of modality.[8] Commonly, modality is cashed out in turns of possible worlds. This prompts the question of what a possible world is. The answers to this question are controversial. We have modal realists, like David Lewis, who endorse a view that possible worlds exist, as real concrete entities.[9] Adopting this analysis for our account of validity would also mean adopting the ontological commitments of this account. Other analyses of modality include modal sceptics, who deny that modal statements can be known. In adopting this approach, we could not know whether our arguments are valid, which is entirely counterintuitive. While there are some more modest approaches to modality, like those taken by Stalnaker[10] and Adams[11], there are still issues surrounding whether these accounts can provide a reductive analysis. This is all to say that while modality is often invoked in philosophical topics, the debate surrounding modal notions is not uncontroversial, and thus any time it is invoked in a theory, that theory faces the same controversies. This is not to say that modal notions should never be used in philosophical theories, but just that we should be aware of the commitment and, all things being equal, adopt theories without modal notions. This gives us a motivation to prefer the interpretational account over the representational account. Indeed, Read, who accepts the representational account over the interpretational account, admits that the lack of modal notions in interpretational account is a possible motivation to prefer this account rather than the representational account.[12]

While this general criticism concerning the use of modal notions is important to note, there is a more specific problem with the representational account; namely, the identification of logical consequence with metaphysical consequence then provides no account of the importance of formality in logical consequence.[13] Similarly, the account does not provide a basis for distinguishing between logical and non-logical vocabulary. This is because the representational account determines that all expressions used in the argument contribute to the validity of the argument. Consequently, the representational account undermines the topic neutrality of logic.

Given that the representational account faces the above challenges, I suggest that this should motivate us to adopt the interpretational account instead. While I do not view these issues as being insurmountable, I simply hold that if there is an alternative we should favour it. If the problem of counterexamples to the interpretational account can be overcome, then this account becomes a preferrable alternative to the representational account of validity. I devote the remainder of this essay to considering and defending a possible solution the interpretational account can adopt to resolve the problem of counterexamples. This solution is the suppressed premise strategy.

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[7] Read, "Formal and Material Consequences", 252.

[8] It should be noted that this conversation concerns analyses of the metaphysical notion of modality, which is distinct from a discussion of modal logic, which is considered to be well understood. Metaphysical modality deals with the fundamental nature of modal notions, whereas modal logic is a formal system which reasons about sentences containing modal operators.

[9] David Lewis, *On the Plurality of Worlds*, (Basil Blackwell, 1986) 2-3, 86.

[10] Robert C. Stalnaker, "Possible Worlds," *Noûs* 10, no. 1, (1976): 65-75.

[11] Robert Merrihew Adams, "Theories of Actuality," *Noûs* 8, no. 3, (1974): 211-231.

[12] Read, "Formal and Material Consequences", 252.

[13] Beall, Restall, and Sagi, "Logical Consequence."

The Suppressed Premise Strategy

The suppressed premise strategy (hereafter SPS) can be employed by the interpretational account to overcome the problem of materially valid arguments. SPS holds that materially valid arguments have suppressed premises which when revealed make the argument formally valid, and thus valid under the interpretational account. These suppressed premises are true given they usually explicitly reveal true analytic connections between words.[14] Since they are true, the addition of the suppressed premise is largely unproblematic, although this claim will be defended further.

SPS applied to the argument (5) gives:

(11) Jill is a paediatrician, all paediatricians are doctors ∴ Jill is a doctor.

This argument can be formalised as follows:

(12) $Fa,\ \forall x(Fx \rightarrow Gx)\ \therefore\ Ga$

There are no possible interpretations of the argument (11) that will have true premises and a false conclusion, thus under the interpretational account (11) is valid, although (5) remains invalid. Of course, this strategy applies to (6), where the suppressed premise is that "taller than" is transitive. No suppressed premise can be added to (9) or (10), since it is not true that all postmen are fathers, there is no analytic connection between being a postman and being a father, and the relation "being friends with" is not transitive.

The Redundancy Objection

The first objection to SPS is put forward by Read and states that the suppressed premise is either false or redundant, and since it cannot be false it must be redundant. [15] Read gives his argument as follows:

> The extra premise is strictly redundant. For if the original argument were invalid, the added premise would not be logically true. Given that it is logically true, it follows that the unexpanded argument was already valid. Hence it was (logically) unnecessary to add the extra premise.[16]

This objection is best demonstrated using an example. Take argument (5), which is considered invalid under the interpretational account. Read says that because of its invalidity, it is possible for the premises of (5) to be true and the conclusion of (5) to be false. This entails that it is possible for Jill to be a paediatrician but not be a doctor. Yet the suppressed premise for this argument is that "all paediatricians are doctors", clearly contradicts the possibility Jill is a paediatrician and not a doctor. It follows if we accept that (5) is invalid, then we also accept that the suppressed premise is false. Yet this suppressed premise is true, so the initial assumption that (5) is invalid must be false, and therefore the addition of the suppressed premise is made redundant for it is not necessary for the argument to be considered valid. According to Read, the suppressed premise's redundancy means we should reject the interpretational account in favour of the representational account.

Read's objection, while presented convincingly, lacks any actual force. This is due to a key error it makes: it presupposes the representational account, when it should presuppose the interpretational account. It is not the case that (5) is invalid because the premise "Jill is a paediatrician" is compatible with it being false that "Jill is a doctor", which (if true) is what the representational account would suppose, rather (5) is invalid because there is an interpretation of (5) for which the truth of the premises is compatible with the falsity of the conclusion. (9) is an interpretation of (5) for which it is compatible that it is true that "Pat is a postman" and false that "Pat is a father", and therefore (5) is considered invalid under the interpretational account. Under the interpretational account, nothing specifically is said about the premises of (5), and so Read is

[14] Read views these suppressed premises not just as true but as logically true because he associates logical truth with analytic truth (Read, "Formal and Material Consequences", 258). Since I have not made this association, I will avoid understanding suppressed premises as logically true.

[15] Read, "Formal and Material Consequences," 257-9.

[16] Read, "Formal and Material Consequences", 259.

wrong to infer that attributing invalidity to (5) will make the suppressed premise false. Since Read is wrong to assert that the invalidity of the argument shows the suppressed premise's falsity, he cannot then infer that since the suppressed premise is true, it must therefore be redundant. Under the representational account, invalidity is saying something about the specific premises of the argument under consideration. Yet under the representational account a materially valid argument, like (5), would not be considered invalid.

Some may reply here that I am begging the question: why is it that we should assume the interpretational account and not the representational account? However, this line of thought is also mistaken. Read clearly starts by assuming that materially valid arguments are invalid, which is only the case under the interpretational account, not the representational account. From this assumption of invalidity, he attempts to prove a contradiction, but then uses the representational account's understanding of validity in this contradiction, even though the representational account would not attribute invalidity to something that is materially valid. However, if the interpretational account is used, then there is no contradiction in using SPS. In addition, this strategy is only used by the interpretational account. Thus, Read must assume the interpretational account if he is going to show a contradiction; given he does not use the interpretational account in his objection and that even if he did use the interpretational account there would be no contradiction, this implies that his objection holds no weight.

Objections about the Nature of the Suppressed Premise and the Argument

A second problem for SPS is that we have not been committed to the view that the suppressed premise is logically true. This may lead to the question: why is it acceptable to add to an argument an extra premise that is not logically true? Surely only logically true propositions may be added to the premises of an argument to retain the same argument. To answer this question, an important point must be reiterated: I do not agree that the argument prior to the addition of the suppressed premise is the same argument as the argument after the addition of the suppressed premise. To me this point is obvious, for the two arguments have different properties: one argument is valid, the other invalid, and they have a different number of premises. Since we are speaking of two different arguments, I do not need to prove that the first argument is "retained" in the second. However, this does not mean SPS can be used on any argument. If the premise "all postmen are fathers" is added to (9) then we have a new argument:

(13) Pat is a postman, all postmen are fathers ∴ Pat is a father.

(13) is a valid argument, but we should not consider (13) to be using SPS. Therefore, we must identify what differentiates (11) from (13), and why (11) is determined as using SPS and thereby linking it closely with (5) in a way that (13) is not linked with (9). The difference is that the suppressed premise revealed in (11) that "all paediatricians are doctors" is true, but the premise "all postmen are fathers" is not true. Indeed "all paediatricians are doctors" is an analytic truth. However, it is not necessary that this be considered a logical truth. To begin with, there seems to be no necessity to consider analytic truths to be logical truths, particularly if we retain the commonly held view that logic has no special content. And secondly, the goodness of an argument can be characterised by whether it is sound, i.e., it is valid and has true premises, which does not require the premises to be logically true. So long as the suppressed premise is true, its addition to the argument does not hinder the chances of the argument being sound and should in fact improve this since the argument will now be formally valid. Since one of the characteristics of a suppressed premise is that it is true, there is no issue that it is not logically true. Considering (13), the premise "all postmen are fathers" cannot be a suppressed premise of the argument (9) for it is not true. Therefore, the suppressed premise does not need to be logically true, but this does not mean that SPS can be applied to any argument to make it valid.

Moreover, we may consider that SPS might even allow us to consider contingent truths as suppressed premises. Let us suppose that it were a contingent fact that "all postmen are fathers", then it might make sense to consider this to be a suppressed premise of argument (9). Say Mr. Black presented argument (9) to Mr. White and both Mr. Black and Mr. White were aware that "all postmen were fathers", then the argument might be accepted as sound in the rhetoric (even though it is not formally valid) because both understood that the argument has a suppressed premise, and that Mr. Black in fact meant to make the

argument (13). Now suppose Mr. Smith questioned the validity of the argument because he was not aware that it was a contingent fact that "all postmen were fathers". Yet, once this would be revealed to him, Mr. Smith would certainly accept the validity of the argument. Therefore, we may accept that a suppressed premise may be contingently true, and it becomes clear that only truth, and not logical truth, is necessary for the suppressed premise.

A counterexample to this argument has been pointed out to me.[17] This is that if we take the argument:

(14) I am a philosophy student ∴ puppies are cute.

This is clearly invalid. But if the conditional "If I am a philosophy student then puppies are cute" is added as a suppressed premise to (14), then we get the new valid argument:

(15) If I am a philosophy student then puppies are cute, I am a philosophy student ∴ puppies are cute.

It appears there is no problem with adding this conditional if we take the view that suppressed premises only need to be contingently true, and not analytically true, because considered as a material conditional it is true (the antecedent and consequent are true). This seems to be a problem for the strategy, as it might allow for many arguments like (14), that have true premises and true conclusions yet are not formally or materially valid, to be valid by adding these conditionals as suppressed premises.

My response to this argument is to say that these conditionals are indicative conditionals, not material conditionals, which means they involve a different treatment. An indicative conditional is the conditional of natural language, and the current discussion is about the validity of natural-language arguments, so it makes sense to speak of indicative conditionals rather than material conditionals. We may then consider views of indicative conditionals which hold that their truth values are different to those of material conditionals, and as such we can formulate a view that holds that "If I am a philosophy student then puppies are cute" is false. For instance, we might hold that an indicative conditional is true iff it is assertable and is in turn assertable iff it passes the Ramsey test. The Ramsey test is a test for the assertability of a conditional, it holds that a conditional is assertable if someone were to add the antecedent to her set of suppositions, she would also have to add the consequent. "If I am a philosophy student then puppies are cute" would clearly fail the Ramsey test. Thus, we can still consider that the suppressed premise may be true without the above presenting as a counterexample.

I have only given a rough sketch of a possible response to the objection suggested above, and while there are many problems with associating the truth conditions of an indicative conditional with those of the material conditional, there are still some who adopt this view. However, the conditional suggested is one where the antecedent and the consequent are both true and yet have nothing to do with each other. This sort of conditional is itself a problem case for someone who holds this truth-functional view of the indicative conditional, suggesting that there is something wrong with equating the indicative conditional with the material conditional. However, if the reader insists on the indicative conditional and the material conditional having the same truth value, even in cases where the antecedent and consequent have no relation to each other, then this reader may simply choose to reject this section on contingent truth and hold that the suppressed premise must be an analytic truth. This does not detract from the fact that the suppressed premise is not a logical truth. Of course, the reader may still object to the idea of analytic truth. However, this paper defends the interpretational account against the counterexample of material valid arguments, which themselves rely heavily on the notion of analyticity. So, if the reader places no importance on the analytic connections between words, then there is no forceful objection to the interpretational account and no need for SPS to begin with.

A third objection connects to my answer to the second objection. I have stated that the two arguments, the argument prior to the addition of the suppressed premise and the argument after this addition, are two different arguments. This may lead one to ask, "what connects the two arguments?" The answer to this is simple: they both have the same aim. The aim of an argument is an imprecise and informal notion; however, I want to use it to capture an intuitive idea. The two arguments share the same conclusion, and their aim is to

[17] By Owen Griffiths, in personal communication.

use true (and very similar) premises to arrive at this conclusion. Suppose that Jones is having a discussion of Jill's profession; he would be just as happy receiving the argument (11) as he would be receiving the argument (5), possibly even happier receiving (11) if he is unaware that a paediatrician is a kind of doctor (or if he is a logician who has a strong appreciation for formal validity). However, Jones would be disappointed if instead of receiving either of these arguments he received (3), for instance, which clearly has nothing to do with Jill or her profession. The aim of the arguments is informal, and the setting for which Jones might accept or reject them, as described, is also informal. The arguments are connected by this informality. The matter of validity in logic is strictly a formal matter, and thus there is a distinct difference between (5) and (11).

The Problem of Modus Ponens

The final problem I shall explore in relation to SPS is the problem of modus ponens. A modus ponens is a deductive argument of the following form:

(16) $A,\ A \rightarrow B\ \therefore\ B$

Modus ponens is discussed by both Read and Timothy Smiley, in very different ways.[18] They both view modus ponens as having a similar form to SPS but speak of different consequences related to this similarity. Below, I address both in turn.

The problem that Read notes with modus ponens is that the major premise of this argument (16) is either false or redundant. While his discussion of this problem is limited, he links it with SPS by arguing that in both cases the additional premise "adds psychological perspicuity [...] But at the same time, it is not essential".[19] To some extent I disagree with both points. Considering the second point, the suppressed premise and the major premise in the modus ponens argument are vital in making the argument valid, and thus are essential to the argument. On the first point, there is some sense in which adding the suppressed premise and the major modus ponens premise do add psychological perspicuity, but it does not necessarily always do this or do this to the extent Read may be suggesting. In cases where both parties implicitly know the suppressed premise, its addition to the argument may not provide any psychological clarity, only logical infallibility. This idea is strengthened when considering that most of the arguments we make in everyday life have suppressed premises and we do not seem to need to reveal these suppressed premises for psychological reasons.[20] Rather we tend to reveal suppressed premises for logical reasons. Given we are holding this discussion in the domain of logic, we may accept the resemblance between SPS and modus ponens while still rejecting Read's assertion of redundancy.

Smiley's discussion of this matter refers to a paradox that seems to be presented by modus ponens and the addition of the suppressed premises. The paradox in question originated from Lewis Carroll, who wrote:

> If I grant (A) All men are mortal, and (B) Socrates is a man, but not (C) The sequence "If all men are mortal, and if Socrates is a man, then Socrates is mortal" is valid, then I do not grant (Z) Socrates is mortal. Again, if I grant C, but not A and B, I still fail to grant Z. Hence, before granting Z, I must grant A and B and C. [Now consider] (D) If A and B and C be true, then Z is true.[21]

This becomes paradoxical when we observe an infinite regress occurring where we must grant (A), (B), (C), (D), and a further (E) If A and B and C and D be true, then Z is true, yet we can think of an infinite number of propositions that must be granted before it seems that Z is granted. We can view (C), (D), etc, as suppressed premises of the argument that Carroll reveals in his paradox. This leads Smiley to comment that "Lewis Carroll was doomed to detect suppressed hypothetical premises even in logically valid arguments, and incidentally to force them all into the straitjacket of modus ponens".[22] If these are considered to be suppressed premises then there is a problem for SPS, for these can be added to any argument, and make the

[18] Read, "Formal and Material Consequences," 259-62; Timothy Smiley, "A Tale of Two Tortoises", *Mind* 104, no. 496, (1995): 727.
[19] Read, "Formal and Material Consequences," 262.
[20] Smiley, "A Tale of Two Tortoises," 727.
[21] Charles Lutwidge Dodgson, *Lewis Carroll's Symbolic Logic*, W. W. Bartley III, ed., (Clarkson Potter, 1977), 472.
[22] Smiley, "A Tale of Two Tortoises," 727.

argument paradoxical. In addition, this does not seem to be what the strategy intends. To solve this, we can examine the characteristics of the suppressed premise, which is that its addition will make the argument formally valid. Yet the arguments that Lewis Carroll imagines are already valid arguments, thus SPS should not be employed in these cases. Smiley's examination of the problem also points out that the specific wording of the paradox is crucial for its paradoxical nature but is itself flawed. Lewis Carroll "lacked any distinct conception of a deduction as opposed to the assertion", and it is this confusion that leads to paradox. [23] By this Smiley means that (C) is not an assertion but a deduction, and so it must be granted, but Carroll seems to think that it is merely an assertion that can be accepted or denied. Hence, this paradox does not show that even valid arguments might have suppressed premises that lead to paradox, thus this objection presents no issue to the use of SPS.

The characterisation I have given of SPS prevents contradiction and I have shown how it is able to overcome objections about the redundancy of the suppressed premise, as well as more generally the nature of the suppressed premise and the nature of the arguments to which it pertains. Finally, I discussed the problem of Modus Ponens, showing two ways it relates to SPS, and that this does not impact the use of the strategy. Thus, SPS is a viable addition to the interpretational account and explains the relation of material validity to validity, without a need to adopt the representational account. Hence by defence of the interpretational account succeeds and preferred to the representational account.

[23] Smiley, "A Tale of Two Tortoises," 727.

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