Contributors:
Marco Dees
Philip Ebert
John Fluharty
Peter Kirwin
Special Contribution:
Prof. Duncan Pritchard

Interview of Marcus Rossberg by
Daniel Paffenholz
Vera Schoeller

Welcome

Its here! After the continuing growth of the philosophy society in the last years, we now not only have more members and events, but finally have our very own journal. We've enjoyed the talks, debates, beach walks and the movie evenings and now openly welcome something that presents itself as a new opportunity for interaction. The articles and interviews included in this issue are intended to form the beginning of a forum of debate between the philosophically minded in St Andrews. We hope future contributions of both advanced and flourishing philosophers will help this journal grow, and serve as a source of mutual inspiration. In the mean time we look forward to a great new year of the philosophy society, hopefully engaging with and making friends with many of you.

Sincerely,
Vera Schoeller and Daniel Paffenholz
President and Vice President

Letter From the Editor

Welcome to Aporia! We have finally got the journal up and running! Our first issue is going to be an excellent one. We have a variety of articles from many philosophers at different levels. Marco Dees presents a summary of his paper on moral responsibility and responding to reasons, which was given at the Burn. Peter Kirwin presents an unusual but excellent paper on Chimera Welfare. There are three papers on Knowledge: Professor Duncan Pritchard of Stirling has contributed a paper on 'The Value of Knowledge', Dr. Philip Ebert has submitted a paper on mathematical knowledge, and there is a paper on the outline of Wittgensteinian 'hinge' propositions from myself. In addition to these articles, Dr Marcus Rossberg has given us a wonderful interview about life as a PhD student.

I sincerely hope everyone enjoys the journal thoroughly and I look forward to receiving future contributions from the undergraduate and graduate community for our next issue!

Sincerely,
John Fluharty

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Credits

Caring for Monsters

PETER KIRWIN

There are five overlapping ethical objections to the creation of chimera: 1) Unnaturalness; 2) Species Integrity; 3) Moral Taboo; 4) Environmental Risk and 5) Chimera Welfare. This chapter will briefly outline the first four, before arguing that considerations of chimera welfare provides us with a strong reason against the creation of some, but not all, chimera. According to the argument from unnaturalness it is simply not our place to intervene in the basic mechanisms of nature. Critics of this view often challenge the coherence of any distinction between nature and human convention and the implicit assumption that nature is perfect as it is . Furthermore, it is often argued that, ad absurdum, the argument leads us to condemn as immoral the damning of rivers and the ploughing of fields.

Sometimes considered a narrower interpretation of the unnaturalness argument, the argument from species integrity representks a realist position in which species are natural kinds whose distinction from each other we are under a moral imperative to respect. Against this view it is often argued that the idea of species is nothing more than a convenient system of classification and that anyway, the mere existence of something does not imply a moral imperative to maintain it .

The argument from moral taboo appeals to a common and intuitive disgust at the creation of chimeras. Such moral taboos, on this account, serve important social functions with taboos against incest, for example, guaranteeing a healthy level of genetic diversity in the community . In response critics counter that few (if any) taboos remain constant across time and cultures, taboos against crossing species boundaries being no exception. Many past (e.g. Ancient Egyptian) and some present (e.g Hinduism) religions contain, or contained, gods who appear to combine, in their form, human elements with those of other species .

TheEnvironmentalRiskargumentappealstoconsequentialconsiderations for the ecosystem and therefore the survival of our species. In making

biological mixtures for which there is no natural precedent we have no idea what to expect. Chimeras may act as half-way houses between species for diseases or, breaking out of laboratories, may breed with other species and upset the delicate balance of the ecosystem. Objections to the argument centre round the extent to which risk can be minimized and the outweighing of such risk by potential benefits .

The Chimera Welfare argument thinks experimentation should be restricted by obligations, both to existing individuals and to those individuals one wants to design and create. Creating some chimeras is wrong because it neglects said obligations. Exactly what obligations do we have to existing sentient individuals? A good place to start is current restrictions, informed consent aside, regarding human test subjects. The "International Ethical Guidelines for Biomedical Research Involving Human Subjects", published by The Council for International Organizations of Medical Sciences (CIOMS) in collaboration with the World Health Organization (WHO), states four ethical principles . "Respect for persons" calls for the "protection" of "dependent and vulnerable" persons "with impaired or diminished autonomy" against "harm or abuse" . "Beneficence" demands that risks be "reasonable" compared to "expected benefits " while "nonmaleficence "proscribes the deliberate infliction of harm on persons ". Finally, "distributive justice" requires "the equitable distribution of both the burdens and the benefits of participation in research " and that difference in distribution be justified by some "morally relevant distinction between persons ". These principles form reasonable restrictions regarding human test subjects who are incapable of informed consent. Indeed, acceptance of such principles appears so widespread that even systematic offenders against them, such as Nazi scientists, did not openly reject them but argued that the principles did not apply because their test subjects were sub-human.

The traditional response to the Nazi scientist is to argue that the test subjects were indeed members of our own species. A more radical approach, however, is to question the assumption that membership of our species is a necessary or sufficient condition to be protected by the CIOMS principles. Why not replace 'persons' with 'sentients' and extend the principles to all sentient creatures? In taking this line I am adopting

Singer's famous call to extend to other species "the basic principle of equality " understood as "equal consideration of interests ". If we oppose the restriction of this principle to one race then, Singer argues, we must oppose its restriction to one species . Any attempt to restrict its extension to ownership of a particular mental or physical attribute is arbitrary because one can always ask, why this attribute and not another ? Furthermore, any attempt to restrict extension to human beings faces the problem that any characteristic basic enough to include every human being will also include many animals . On the other hand, any attempt to choose an attribute that will exclude these animals will also exclude many human beings like young infants and severely mentally disabled infants or adults . Singer dismisses out of hand, tying the principle of equality to the simple fact of species membership on the grounds that this is analogous to tying it to membership of a particular race . The capacity for "suffering" as a necessary and sufficient condition for having interests is, for Singer, the only reasonable boundary .

A large number of those who wish to limit the basic principle of equality to our own species do this on the grounds of a special "human dignity" not found in other species . This concept has its roots in the Judeo-Christian tradition and is traditionally justified by man's position in the "great chain of being" where he has "dominion" over animals . Other justifications include souls, a likeness to God and Jesus' Incarnation in a human body . Secular versions of human dignity have their roots in Kant's avowal that only "rational beings" have "unconditional and incomparable worth" or dignity because they are capable of moral agency . Unfortunately, modern secular conceptions often never define 'human dignity' , and so leave us wondering why it is not extended to animals, or define it in such a way that many arbitrary qualifications are needed to avoid the exclusion of infants and severe mental defectives. Conceptions of human dignity can detract from animal welfare by either implying that, in a conflict of human and animal interests, it is morally permissible to favor humans or by suggesting that only human interests are morally relevant.

Human dignity aside, an alternative response to our position is to consider, as does Michael Allen Fox , the moral community in terms of a social contract model. Building upon Kant's stress on moral agency, Fox

argues that moral obligation can only exist within a moral community consisting of a series of "mutual guarantees", "by tacit agreement", of "nonintervention in the self-governing lives of others ". However, to participate in a mutual guarantee one must be able to understand and implement one's own side of the bargain. In short, to be the subject of moral obligations you must be capable of moral agency. To be a moral agent one must be autonomous which, for Fox, requires not only "critical self awareness; the ability to manipulate complex concepts and to use a sophisticated language " but also "the capacity to reflect, plan, deliberate, choose, and accept responsibility for acting ". Not being autonomous, animals are incapable of moral agency and thus are not subjects of moral obligation. Given its necessity to participation in any contract and its description in terms of a cluster of features, Fox's stipulation of moral agency appears to avoid Singer's usual charge of arbitrariness.

Its seems, however, that Fox's argument still sets the bar for membership of the moral community so high that many normal infants, severely mentally defective infants and adults are denied membership. Fox's reply is that normal infants are brought under the protection of the moral community because they have autonomy in "latency ". It is briefly suggested that severely mentally disabled adults and infants should be protected because this provides a form of insurance for moral agents should they, through illness or injury, lose their own autonomy . Putting aside, for charity, problems with the moral relevance of potential in normal infants, the idea of insurance works well for adults who have lost their autonomy through accident or illness. However, Fox needs to find some other way of justifying moral obligation towards severely mentally deficient infants which have no 'latent' autonomy and whose protection is not prudential for moral agents: a forty-year-old man is not afraid of becoming a mentally deficient four-year-old. Fox's answer is two-fold. Firstly it is argued that our intuition in favour of moral preference for members of our immediate family justifies moral preference for our "human family ". For additional support, Fox adopts John Passmore's account of a "chain of love and concern " that extends down the generations and includes the "places, institutions and forms of activity " that constitute our day-to-day existence. Though not explicitly put, the thought seems to be that moral agents care for their descendents who may turn out to be mentally defective infants, therefore the protection of

mentally defective infants is in fact another form of prudence.

The first part of this argument will not wash. Fox needs to assume that the original intuition is based on our biological relation to immediate family members, in which case the intuition would also justify racism. However, the intuition is based upon the relationships built with family members (natural or adopted) and thus does not extend to strangers from our own species. Furthermore, the intuition justifies moral preference regarding superogatory acts as opposed to basic moral obligations . It will justify buying family members, but not strangers, birthday cards but will not justify stealing from strangers. Fox's second argument needs to take account of the fact that, given the progress of biotechnology, one of my distant descendents may be genetically altered so that moral agents do not consider the resulting chimera wholly human. Given this, it seems that concern for one's descendents must in fact lead us to extend "the basic principle of equality " to all sentient creatures.

Having established our moral obligations to existing organisms the question arises as to what obligations we have to organisms that will come into existence as a result of our design and creation. In answer to this I would like to adopt a slightly altered version of Bernard Rollin's "Principle of Conservation" which states that it is not morally permissible to bring into existence a creature whose expected quality of life (as a result of the developmental modification) is likely to be lower than is normal for the host's parent stock . If an alteration in functional capacities lowers quality of life below this point it is most likely due to the frustration of telos. Telos is an originally Aristotelian concept, according to which each species of animal has a natural way of life consisting of a series of ends or activities, some of which are shared by other species and some of which are species specific . Individual organisms are instinctively driven to fulfil these, with success in the enterprise creating contentment and failure creating suffering. If our previous argument about extending "the basic principle of equality" is accepted then the Principle of Conservation applies to all possible sentient creations.

Before moving on let us deal briefly with some objections that may be made to The Principle of Conservation. One objection is that a creature must exist before it can be harmed or benefited and so the act of

creation itself does neither good nor evil to the organism in question . It may be responded that this objection rests upon the broad assumption that all moral wrongs involve wronging individuals when there are certain things, such as destroying the last instance of a rare orchid, which we consider wrong independently of harm to individuals . Against this it will likely be countered that such 'victimless wrongs' still derive their wrongness from effects upon individuals. If I destroy the orchid many people will lose the chance to experience its beauty while my contribution to the lack of biodiversity harms the environment and thus all individuals living in it. To truly silence the objector it is tempting to argue intuitively using a thought experiment in which, when crushing the orchid, I am the last sentient individual in the universe. Such refutation is not necessary, however, for the key problem with this objection is its implicit assumption that actions can only be wrong due to effects upon "identifiable individuals ". Parfit , however, recognises that such a view would commit us to regarding many future actions, such as setting a bomb under St. Salvator's quad to go off in fifty years, as morally neutral because they don't harm or benefit "identifiable individuals ". Many of the victims are unidentifiable because their identities will be the result of decisions made between now and then but it seems absurd to say that they cannot therefore be harmed.

It may also be objected that measuring quality of life to any useful degree involves an understanding of the consciousness involved and, as such, is obviously impossible when that consciousness does not yet exist. In answer to this I take what Degrazia calls an objective view of wellbeing according to which judgment of the future organism's quality of life is based upon her chances of achieving "species-typical levels of mental and physical functioning ". This approach maintains a subjective element, however, insofar as the degree to which the future organism is likely meet these standards gives us some idea how good or bad life will be from its own perspective.

Accepting the Principle for the Conservation of Welfare, it follows that, when our normative principles are combined with the definition of chimera reached in the first chapter, we will judge the moral permissibility of creating chimeras on a case-by-case basis. Chimeras resulting from intervention in the embryo (pre-differentiation) will be judged by the

New Principle for the Conservation of Welfare and in doing so we must understand three crucial points. Firstly, not all new functional capacities change telos; secondly, those which do not may either help, hinder, or not effect fulfilment of the existing telos and thirdly, relative to physiology, changes in the telos may either preserve or diminish welfare. That some alterations in arrangement do not change telos is intuitively obvious given that telos is psychologically determined and, for example, whether or not an individual has wings will not affect her underlying psychology. Such a change would not change telos but does provide us with an example of a non-telos-changing alteration in arrangement that would help the organism fulfil its current telos. Accomplishing one's ends or goals would be a lot easier if one had wings and the same would seem to apply to individuals with, say, sonar or night vision. An example of a non-telos-changing alteration in arrangement that would not affect the organism's ability to fulfil its telos might be an 'omnicow': a cow who can also digest meat. Such an alteration would have no effect upon telos because, while it does not interfere with the normal activities of a cow, neither does it give the omnicow an advantage in fulfilling her ends. On the other hand, one non-telos-changing alteration in arrangement that would probably hinder the fulfilment of telos might be a dog with a shark's tail in place of its two back legs. This 'sharog' would seriously struggle to accomplish its ends upon land and, though the tail might help him swim a little faster, this is of little consequence, there being nothing in an average dog's telos that requires swimming. Those deviations in a chimera from the standard functional arrangement of its species which produce a fundamentally different underlying psychology may change the telos for good or bad. A change in telos is a change for good insofar as something has been added to the original telos and it is ensured, perhaps through simultaneous changes in physical arrangement of the organism, that the organism is capable of fulfilling the new aspects of its telos. To use a concrete example, intervention in a cat embryo to create a catman with the desire, in addition to a normal cat telos, to communicate with language is permissible insofar as the catman's design includes whatever physical augmentation is necessary to use a complex language.

Regarding chimeras resulting from xenografts of large parts of the brain in post-differentiated embryos or adults, these will be judged by our version of the four CIOMS principles which implicitly include considerations

of telos. To use a concrete example, let us examine the quail-chicken chimeras referred to in chapter one. This case appears to violate at least half of our principles for research on those incapable of consent. The exploitation of the chickens is contrary to both 'distributive justice' and the principle of 'beneficience' because there is no equal distribution of the burdens and benefits of participation and the chickens involved are put at great risk while appearing to gain nothing from the experiments. Whether their creation is contrary to 'respect for sentients' and 'non-malefience' will depend on whether the quail chicken's biology allows it to fulfil whatever quail ends it is conscious of . Given that chicken and quail physiology appear broadly similar one would guess that it would be able to fulfil these but, pending an exact comparison, we reserve final judgment.

Creating a chim
era from a brain xenograft, we might think, would be permissible if it made the host far more intelligent. Here we should be cautious, however, and remember that intelligence by itself does not increase welfare and may actually reduce it. Not only can intelligence increase suffering through an increased knowledge of one's situation but also, as we have seen, by adding something to the telos which the body is incapable of satisfying. If a xenograft designed to increase intelligence changes the telos of the chimera in question from the parent stock, the same rule applies as to the humanzee created by developmental modification. Telos may only be expanded if it is combined with physical augmentation which ensures that the new telos can be fulfilled. If this condition, along with our version of the four CIOMS principle, is satisfied then the sentient in question could benefit from a brain xenograft and the creation of this chimera might be permissible.

Having established that creation of certain chimeras is wrong, the question arises as to how this 'wrongness' is to be interpreted. To one extreme we might mean wrong in a strict deontological sense where it is always impermissible and no room is allowed for mitigating factors. To the other extreme we might mean wrong in a consequentialist sense which allows that wrongness to be outweighed by other consequences in the pursuit of a greater good or the avoidance of a greater evil. The key question for whatever sense we choose is whether the means can ever justify the ends or, to be more specific, whether the creation of such

chimeras might be justified if they led to extremely valuable medical advances. There is not space here for a full answer to this question and so it will suffice to leave the reader to make up her own mind on this. In doing so she should, however, be cautioned that the fact that our normative principles centre upon negative duties not to inflict pain and suffering does not necessarily imply as a bedrock a Utilitarian system which includes positive duties to increase the overall welfare of all sentient creatures.

Our objection from Chimera Welfare argues that we have obligations both to existing sentient individuals and those individuals we bring into existence by our creation and design. These obligations are extended to all individuals (sentient individuals in the first case) regardless of species. Two sets of principles constitute the content of these obligations. Firstly, our version of the CIOMS principles prohibit, in the case of existing sentient creatures, intervention where any benefit to the creature is outweighed by its harms and burdens. Secondly, the Principle for the Conservation of Welfare prohibits the creation by developmental modification of any chimera whose expected quality of life will be lower than that of the host's parent stock. Given limitation of length, the nature of these prohibitions – whether they are absolute and, if not, in what scenarios they might be overridden – is here left to the judgment of the reader.

Survey of Modern Wittgensteinian Hinge Theories in Epistemology

JOHN FLUHARTY

The following essay is a survey of uses of hinge propositions in epistemological theories. I shall begin by looking at the sceptical paradox which is the main antagonist in Epistemology. I will then move on to look at the origins of the idea of hinge propositions, which had their root in late Wittgenstein. Finally I shall look at the two most prominent modern theories utilizing hinges; those of Crispin Wright and Michael Williams. In addition, I shall cover some of the main problems concerning these theories, and some brief possible responses.

1.0 The Sceptical Paradox

There are many forms of the sceptical hypothesis (SH), but perhaps the most common, is the BIV argument:

(1)   It is possible that I might be a BIV being fed all my 'experiences' of the world.
(2)   It is impossible definitively to determine that I am not a BIV.
(3)   If I cannot definitively determine that I am not a BIV, then most, if not all, of my beliefs lack sufficient epistemic status.
Hence:
(C) Most, if not all, of my beliefs lack sufficient epistemic status.

What we see as a consequence of the sceptical argument is the incompatibility of these three claims:

(S1) We cannot know the refutations of SH
(S2) If we are unable to know the refutations of SH, then it follows that we do not know 'everyday' propositions.
(S3) 'Everyday' propositions are impossible to know

Following from (S2), knowledge is 'closed' under known entailment. Primarily known as the closure principle:

Closure for Knowledge: For all agents, φ, Ψ, if an agent knows a proposition φ, and knows that φ entails a second proposition Ψ, then that agent also knows Ψ.

However, because closure permits (S2), it allows the sceptic to deny (S3) on the basis of (S1). The main force of this argument, which I wish to convey, is that due to the sceptic, we are unable to know everyday propositions.

1.1 Origin of Hinge Propositions

The idea of hinge propositions was originally published in Wittgenstein's final book, On Certainty, as a response to Moore's objection to scepticism:

(1)   I know that I have two hands.
(2)   If I know that I have two hands, then I know that there is an external world.
Hence:
(C) I know that there is an external world

Wittgenstein, drawing much from Hume, formed the idea of 'hinge propositions', in which there are particular propositions that one may believe but in addition, one may exempt from doubt. It is the belief in these particular propositions that enables one to begin one's scientific investigations. They are not supported by reasons. The propositions are indeed the framework on which such investigations begin. For, 'If I want the door to turn, the hinges must stay put.' The sceptic, then, mistakes a hinge proposition for a normal (non-hinge) proposition, when she asks for reasons or justification. Taken in this light, hinge propositions can be considered to be propositions which one does not have direct grounds to believe, though they are believed, nevertheless because of their important role in scientific investigations or methodology. In Peter Strawson's words:

The correct way [of dealing with] with the professional skeptical doubt

is not to attempt to rebut it with argument, but to point out that it is idle, unreal, a pretence; and then the rebutting arguments will appear as equally idle; the reasons produced in those arguments to justify [our beliefs ...] are not, and do not become, our reasons for these beliefs; there is no such thing as the reasons for which we hold these beliefs. [...] We simply cannot help accepting them as defining the areas within which the questions come up of what beliefs we should rationally hold on such-and-such a matter.

The concept of a hinge proposition entails, for Moore, that he (perhaps like the sceptic) mistakes a hinge proposition for a proposition that is empirical. More explicitly: in claiming to know P, subject S implies that empirical grounds can be offered in support of P, and this justification is more certain than P. However, since there is no empirical evidence for Hinge Propositions, there can be no justification in resting on them.

Wittgenstein argues, then, for the revision of our understanding of the structure of reasons and empirical evidence, consequently implying that an assertion of a hinge proposition in the first-person is not conversationally apposite. This shows parallels to foundationalist epistemology, because the foundations of hinge epistemology are both self-justifying and not justified by anything else believed by a subject. Importantly, this also entails that doubts of hinge propositions are groundless, which means both the Moorean and the sceptic are misled by their claim to know and doubt, respectively, hinge propositions.

I have been dealing primarily with a minimalist interpretation of Wittgenstein's remarks on hinge propositions, and, for the anti-sceptic to denounce the sceptic, a strong thesis of epistemological hinge propositions must be constructed for three reasons:

1)   the minimal thesis tells us next to nothing about what the conditions for knowledge are, focusing instead on what the appropriate conditions are for claims to know (or claims to doubt).

2)   the minimal interpretation of On Certainty, if understood as an anti-sceptical thesis, makes the mistake […] of treating the sceptic as an embodied adversary, someone who is committed to making appropriate

claims about their epistemic position (and ours).

3)   the minimal interpretation seems to favour a sceptical reading of our epistemic predicament over an anti-sceptical reading.

From here, I shall take these reasons into consideration and in section 2 move onto the more modern utilizations of hinge propositions.

2.0 Modern Hinge Theories

Under Richard Miller's interpretation of hinge propositions, 'epistemic rationality'—or one's rationality that allows for the most advantageous number of true beliefs—and hinge propositions can be used interchangeably. Thus, to believe in certain propositions may enable an individual to optimize their number of true beliefs even though the beliefs may not have reasons. Therefore, one can believe in a hinge proposition, which is groundless, but nevertheless, allows the individual optimal true beliefs. Miller's argument may be summarized as follows:

1)   There are no grounds to support hinge propositions and we are compelled to believe them

2)   Belief in hinge propositions is necessary for true beliefs. Hinge propositions are "epistemically indispensable resources".

3)   Belief in hinge propositions are not detrimental in any form because belief in them involves no cognitive apprehension, and thus trust in them cannot lead away from a belief. No new evidence can count for or against a hinge proposition.

Therefore, it is epistemically rational to believe in hinge propositions, This theory faces a number of problems though, the most difficult of which is that the sceptic can simply propose an epistemic reality which does not entail belief in hinge propositions.

2.1 Wright's Theory

The subversion argument may be able to be saved, though, by Crispin Wright, whose anti-sceptical thesis originated from his arguments on 'McKinsey paradoxes', which involve the incompatibility of first-

person authority and semantic externalism . The logical structure of McKinsey paradoxes mirrors that of sceptical arguments, in that first person authority and externalism produce an implausible conclusion. The paradox moreover involves knowledge transfers across known entailments, which assume closure for knowledge. Wright argues that closure cannot be rejected, but rather rejects a similar principle of 'transmission'. Closure simply holds that knowledge transfers across known entailments, where transmission is stronger, insisting that 'what grounds the agent's knowledge of the antecedent proposition thereby grounds the agent's knowledge of the consequent proposition' . Closure does not demand the same kind of stress on the antecedent proposition. Wright summarizes the point as follows:

A valid argument is one thing. A valid argument with warranted premises is a second. But a cogent argument is yet a third: it is an argument, roughly, whereby someone could/should be moved to rational conviction of the truth of its conclusion—a case where it is possible to learn of the truth of a conclusion by getting warrant for the premises and then reasoning to it by the steps involved in the argument in question. Thus a valid argument with warranted premises cannot be cogent if the route to warrant for its premises goes—of necessity, or under the particular constraints of a given epistemic context—via a prior warrant for its conclusion. Such arguments, as we like to say, 'beg the question'. Say that a particular warrant, w, transmits across a valid argument just in case the argument is cogent when w is the warrant for its premises .

Thus, one can be warranted in believing the antecedent proposition, without being warranted in believing the consequent; moreover, the grounds for belief in the antecedent with a known entailment to the consequent does not entail that there is a warranted belief in the consequent. This ultimately results in Wright's proposal that there are hinge-like propositions that one is entitled to believe but for which one cannot suggest any evidence, which are specifically the anti-sceptical propositions. Closure is not the problem here, but transmission of warrant.

Why is it, however, that Wright is able to obtain this anti-sceptical conclusion rather than a sceptical conclusion that we cannot know

these hinge propositions, and if closure holds, then we know little, if anything? This regrettably is a difficult problem, though one can argue that even though hinge propositions have no grounds for our belief, we are warranted in believing them, as not believing in hinge propositions would result in most of our knowledge being called into question. Wright, though, is an epistemic internalist—'adequate reflectively accessible supporting grounds are essential to the possession of warrant or knowledge (at least as regards propositions which, like hinge propositions, are not "self-justifying" in the way that foundational propositions are taken to be on the classical internalist picture—i.e., by being self-evident, or incorrigible, and so forth)' .

We now see that unearned warrants, or those without supporting reasons, are the main problem for Wright, as according to internalists, those warrants are not genuine. Though Wright's response is that there are reflectively accessible grounds to support such hinge propositions, these reflective grounds are not epistemic, and thus, via internalism, they are not warrants at all. Wright's distinction between transmission and closure never contributes to the argument as according Wright's argument 'Knowledge is essentially to do with the possession of reflectively accessible reasons, and the structure of reasons is such that our believing is ultimately groundless' . Thus, Wright's theory is in danger of falling into a form of scepticism. In addition to these reservations about unearned warrants, I propose an additional reservation dealing with the structure of Wright's argument. Wright asserts that we must have a warrant for the conclusion of an argument, where transmission is a problem, but says that this warrant comes from somewhere else, specifically hinge propositions. Firstly, looking for warrant outside of a philosophical argument, I think, is in some form endorsing externalism, for if one cannot internally reflect to obtain warrant, then the warrant must come from an external source. Secondly, as hinge propositions have no reasons to support them, could they in fact be externalist elements? Hinge propositions are notions of the external world which are unknowable to us, but none the less are important in reasoning. Finally, if indeed, Wright is resting on elements that are externalist in nature, one might ask who declares that the person in question has a warrant for his belief. If hinges do not have reasons for us to rest our philosophical inquiry on, then surely someone must declare that an

individual is warranted in believing this hinge proposition. Wright's theory seems to rest on the subject, but would it also be important to ask whether an attributor would be in a better position?

Perhaps, what Wright needs is some justification for believing in hinge propositions beyond the mere assertion that, without them, our branching theories would fail to work properly. For example, in physics, there are many theories that have mathematically explained the workings of certain aspects of the universe (i.e. gravity) but which have been revised numerous times with the acquisition of new data. Thus, I believe it is not acceptable merely to stress that hinges are existent based on the problems we would encounter, were we not to believe in them. I will, though, set these thoughts aside, and move on to another epistemic theory utilizing hinge propositions.

2.2 William's Theory

Williams looks to rectify Wright's problems by turning to an externalist view of hinge propositions, which also draws from newer inferential contextualism; which differs from subject contextualism (put forward by Keith DeRose) where the standards of knowledge are set by the conversational contexts of the attributor's context, but rather the subject's context sets the epistemic standards. In each context, there will be epistemic standards that cannot be challenged, and the rest of the beliefs in the epistemic context will be tested relative to the hinge propositions held in that particular context. However, Williams claims that different beliefs in different contexts play the role of a hinge proposition, which reflects movement between the epistemic contexts.

Williams's thesis separates itself from other contextualist theories on these claims:

1)   "Methodological Necessities" are held to be true in the want of any supporting grounds.

2)   There is no 'hierarchy' of contexts.

Semantic contextualism generally states that sceptical hypotheses are more demanding then 'normal'. However, Williams asserts that all contexts are separate from one another. A sceptical context has the hinges

of an assumption that knowledge is context invariant, i.e.: knowledge of the world is unattainable under philosophical investigation. Both of these claims appear controversial to me, as certainly necessity is always controversial in post Quinian philosophy, but more troubling is Williams's lack of a hierarchy of contexts. Certainly there are contexts, such as a philosophical one or sceptical one, which are more pressing, and thus involve more justification or pedigree. Williams, though, retains closure: normal contexts allow subjects to know hinge propositions presupposed. However, the main problem faced by Williams here is one that plagued DeRose and Lewis as well, in which any mention of the hinge propositions changes the epistemic context to a philosophical one, in which different hinges would be found to hold. In the sceptical context, the Sceptical Hypothesis (SH) holds, though Williams claims that some anti-sceptic propositions are unattainable in sceptical contexts but he holds closure, allowing for these propositions to be known in normal contexts. Thus a subject can have normal knowledge and have knowledge of denials of SH.

Williams's thesis, though, has more problems: most notably perhaps is the fact that subjects know tacitly the context's hinges due to the externalist nature of the thesis. Moreover, Williams is moving to reject epistemological realism, but merely asserts that it is ambiguous. According to Williams, realism is embedded in sceptical contexts, and thus we should ignore the sceptical context all together. In the sceptical context there is not only a lack of justification, but also there is a case for beliefs to be false. However in the normal contexts, only the former holds. However, Williams, at times, wants to hold that there are true conclusions relative to a sceptical context.

Williams's problems with his theory, namely the problem of not being able to assert one's knowledge (which is the same problem plaguing other forms of contextualism), leads me to believe that the incorrect move, for Williams, was to endorse contextualism. However, Subject Sensitive Invariantism does not deal with the sceptical problem very well, I feel perhaps an SSI theory that introduces hinge propositions may prove more effective. In SSI, one's warrant for belief changes with context, instead of one's knowledge. Thus, perhaps we can have knowledge of many things, however warrant changes with the context, diminishing

the amount of things we know in more challenging contexts (perhaps representing the further one goes from the basic hinge propositions). In other words, the closer to basic hinges we are, the more warrant we have for believing a proposition, but the more or complex the belief, the more warrant needed, as it is further from the hinge.

This essay has merely been a survey of the recent work in epistemological discipline on hinge theories. All I have attempted to show is that the modern theories do in fact face many problems, but there, perhaps, is hope to challenge the sceptic by combining the hinge theories with other modern disciplines in epistemology, such as Subject Sensitive Invariantism, or other forms of Contextualism.

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5. McKinsey paradoxes argue that first-person authority and semantic externalism are incompatible because combining the two results in a conclusion that one can have a priori knowledge of empirical facts.
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War and Paradise, An Interview with Marcus Rossberg

INTERVIEWERS: VERA SCHOELLER AND DANIEL PAFFENHOLZ

Marcus Rossberg has been in St Andrews since 2001, finished his PhD in March 2006, and is now a postdoctoral research fellow here. His interests lie in philosophy of mathematics, philosophical logic and metaphysics. Thanks to this, he had some illuminating advice on career prospects, as well as sharing his insight on the differences between continental and analytical philosophy and other battlefronts. For more information about Marcus Rossberg, check his website at:
 http://www.st-andrews.ac.uk/~mr30/

Vera Schoeller: We looked on your website and saw that you've studied art history. When, how and why did you make the switch to philosophy?

Marcus Rossberg: I did a double major in philosophy and art history, so I studied both right from the beginning. I wrote my master thesis in art history, and after finishing it I knew that I wanted to philosophy. (laughs)

V: What was it about philosophy that inspired the switch?

M: Art History just wasn't the right thing for me. I discovered that what I like about philosophy is the rigorous thinking. I don't want to say that you can't practise art history in this way. It is just that at the time in Germany art history wasn't often done with analytic rigour. I thought that if I want to have a career in academia then I want colleagues who have more or less the same idea about how to do research that I have. This, basically, means adopting a good and rigorous methodology, like in the sciences, also in the humanities.

Daniel Paffenholz: Did you face any challenges when you told your

friends and family that you want to do philosophy?

M: That had already happened when I said I wanted to do art history! After that, I guess, everyone thought I was lost anyway, so it didn't really matter anymore.

V: So you had no problems answering the questions about what philosophy would do for your future?

M: I wanted a career in academia. Nobody in my family is an academic, so they didn't really know what to think about all this, and just took my word for it.

V: So why St Andrews?

M: Towards the end of my undergraduate studies I became interested in the philosophy of mathematics. I started my PhD in Düsseldorf, where I did my undergraduate degree. Then the Arché Research Centre announced scholarships for PhDs in that area of research, the philosophy of mathematics. So I thought, ok, there are lots of people there working in philosophy of mathematics, and there are next to none in Germany, or at least none in Düsseldorf. Moreover, I've got five or six books on the philosophy of mathematics in my bookshelf, and the names on their covers are names that you also see on the staff list in St Andrews. So I thought this would be a good thing to try: it can't hurt to apply.

D: Is that also how you chose the department in St Andrews?

M: Yes. I thought they're offering scholarships and all the people who have got big names in the philosophy of mathematics are here, or are regular visitors, so it's a really good department. It's practically the place in Europe where you want to be if you want to do philosophy of mathematics! So I tried it and got lucky.

V: Did you notice a huge change between the way philosophy is done here and the way it is done in Germany, in terms of analytical versus continental philosophy?

M: Oh yes! (laughs) In Germany there is an ongoing struggle between continental and analytical philosophy. It's really a bit like war. Here, it is just natural that if you do philosophy you do analytic philosophy. It was like coming to paradise!

V: In Germany, is there a lot of interaction between the two approaches, or is it like two different languages?

M: Well, I tend to think that there is only one school of philosophy, and then there is something else, which is called continental philosophy.

D: You've been in St Andrews too long!

M: No, I thought that before I came here. And I actually observed that I and some of my German friends, who are also analytic philosophers, are probably a bit more radical concerning analytical philosophy, just because we were in the war. We fought that battle, and you don't have to do that here. To many people in the UK and the US it seems acceptable to do a little bit of postmodern philosophy on the side. I just don't think so, because it is all rubbish.

D: Coming back to the career topic, can you give any good advice for undergrad philosophy students when they're making career choices?

M: It is hard to get a career in philosophy; it is very difficult. You have to be pretty determined, and know that this is really what you want, that this is really what you enjoy doing. Because if you are clever enough for philosophy you are probably also clever enough for most other things – obviously, because philosophy is one of the hardest subjects. And if you are successful in philosophy then you can be successful in most other jobs, where you would earn much more money. Way more money than you will ever earn in philosophy. So, beyond your undergraduate or second degree, you have to be pretty sure that philosophy is really where your passion lies.

V: Do you miss your undergrad days?

M: No.

V: What are the benefits of going for a PhD?

M: You mean besides it being necessary for an academic career? If you start doing a research degree, that is, an M.Phil or PhD, then you don't have to take any courses anymore, so you can really concentrate on the area that you are interested in. The system is different in Germany and I didn't do my undergrad here, so I'm probably not the best person to judge, but there are a lot of courses in undergrad that you have to do – and that's a good thing because you need the background. But you need to study areas that you're not really interested in and that can be tedious, as you know. That's all behind you once you start a research degree, and that feels great! Obviously if you're doing a research degree you're just doing research, and only in the aera that interests you. Then you do your PhD for three, four, five years, and towards the end you get really sick of the subject, but a couple of moths after you handed it in everything is fine again, and you love your subject again.

D: And the social side of undergrad studies compared to postgrad studies?

M: The PhD group in St Andrews is fantastically close. So, socially it has been much better than my undergraduate. On the other hand, I don't know what it is like to be an undergraduate in St Andrews. But the postgraduate student community is good, and PhDs and a lot of the M.Litts spend their free time together. It's a really a fantastic community – so, that was great. But I hear it's not like that everywhere, that St Andrews is special. Probably because there's nothing else to do around here.

V: On that note, what does your philosophical day look like?

M: (laughs) I have got this research postdoc, which means that I don't have any teaching obligations, and most of my time is supposed to be dedicated to my own research: writing my own papers, which of course after finishing a PhD will most likely be taking chapters out of your PhD thesis and turning them into papers, sending them off to journals. This is what I do at the moment. Now, the job that I have concerns the translation of Frege's Basics Laws of Arithmetic, so its not your typical

philosophical research. Philip (Ebert) and I spend a lot of time on the translation, of course, since this is our job. Say 50 % has to do with translation and 50% with our own research. Now there is quite a lot of administration that needs to be done as well: correspondence with people, getting research seminars running, organising conferences. That sort of stuff. You always think that you can do that on the side, but it actually takes a lot of time out of your day. So I guess probably almost 50% of my time is administration at the moment, mainly due to the conference that Philip and I are organising.

D: Do you need to be in a certain mood to do philosophy?

M: I find it hard to do philosophy in the morning. Its not that I have problems to concentrate in the morning: I can do other productive things, like all the admin. I quite often do philosophy at night. In the morning I'm probably too energetic to sit down and think carefully.

V: Do you have something that inspires you if you're not in the mood?

M: No. (laughs) Its really not so much a mood thing, its just in the morning I'm not patient enough.

V: Does it have anything to do with certain philosophers?
D: Are there any philosophers you hate most?

M: Yeah, but I don't deal with them. They are all the continental philosophers! I can give you an alphabetic listing! French poststructuralists are pretty high up on my hate list. Like Lacan and Derrida and these people. More famous people on my hate list are Heidegger and Hegel.

V: Have you changed any of your moral stances and codes since studying philosophy?

M: No. Well I'm not doing moral philosophy, that's one thing. Well, I don't know..(laughs)

D: Do you ever have any existential doubts?

M: No It is also difficult to come across something like that if you're doing philosophy of mathematics!

V: Do you think a career in philosophy should come with any warning tags?

M: Well yes. As I said earlier, if you're good enough to do philosophy you're probably good enough to earn loads of money doing something else. So you really have to be dedicated. Its going to be pretty hard and you will have to work pretty hard. You probably have to spend more hours doing philosophy than you would spend in an ordinary nine-to-five job. And if you're absolutely sure this is where your heart lies, then you should do it.

D: We heard that there's a football game between Logic and Metaphysics, and Moral Philosophy in the department. Did you participate in the last game?

M: The Edgecliffe Cup! I'm on the logic and metaphysics side, and I really want logic and metaphysics to win, so I decided not to play. It has always been the case that moral philosophy won, in the past however-many years, except for one rare occasion, I think five years ago. But this year logic and metaphysics won for the first time in many years. It was a quite high score as well. I am not quite sure, something like 11 to 2.

D: Who scored?

M: I don't recall, but I suppose Philip was pretty good. Actually, we should go over to Philip's office after we're done here. I'm sure you can get that information out of him! (laughs)

(Actually, the score was 8:1 for Logic & Metaphysics. On the Moral Philosophy side Brian McElwee scored a penalty; for Logic & Metaphysics, Philip Ebert scored 4 goals, Paul McCallion scored 2, and Geoff Wright (one of Crispin Wright's sons) and Doug Edwards score one goal each.)

V: Any pubs around St Andrews you highly recommend?

M: The Cellar! That is, Aikman's, I think it's now the only place really I like. We also used to go to the Castle before it was refurbished. There's a philosophy postgraduate research seminar every Friday, from 4 to 6, and we always went to the Castle together immediately afterwards. They had a happy hour and it was great! There was also always enough place for all of us. And now it has been refurbished, it's horrible, so we can't go there anymore. I suppose this is an opportunity for the new postgraduates to find a new place to hang out. I'm a big fan of the Cellar, personally.

(Summary of) Paper given at the Burn: Moral Responsibility and Responding to Reasons

MARCO DEES

One prima facie plausible version of compatibilism regarding moral responsibility and determinism holds that an agent is morally responsible in performing some action just in case this action is performed for a reason. (See e.g. Fischer & Ravizza 1999). Intuitive enough: the account rightly says we aren't responsible for what we do if we are hypnotized, or if we are robots, or if we don't intend to act. Free will is important because it is important that we can conceive of ourselves as moral agents. So this is an account that, since acting for a reason is a way of being caused, rescues the important part of agency – moral responsibility – from the threat of determinism. But we shall see that any such account is problematic.

Intuitively, even if someone takes a reason to justify an action when it doesn't, there is still a sense in which he acts for a reason. Distinguish between justifying reasons and motivating reasons: motivating reasons are psychological states that explain behaviour. (It seems we can have a motivating reason to do one thing, but since we have a stronger reason to something else, not act on it. So motivating reasons are potentially explanatory.) (See Michael Smith, 'The Moral Problem' for more on motivating reasons)

What counts as an acceptable explanation of a fact depends on what the fact is contrasted with. Consider the question, "why did you go to the opening ball?" What will explain effectively will depend on the contrast: "why did you – rather than someone else – go to the ball?", or "why did you go to the opening ball – rather than some other ball?", or "why did you go to the opening ball - rather than just staying at home?" (For accounts of 'contrastive' explanation, see David Lewis, 'Causal Explanation' 1986 or Peter Lipton, 'Inference to the Best Explanation'

1991: both give accounts of how explanations of a contrast must give a causal difference between them of a certain form.) The context determines which of these contrastive questions is referred to by the bald question, "why did you go to the opening ball?"

If an action is to be morally responsible it must be performed for a reason, in the sense that there is a reason that can figure as the explanation of that action. But as we saw, what counts as an explanation of a fact depends on what the fact is contrasted with. So take some act, say the helping of an old lady. Was it responsible? Depends on the context and the contrast specified. Why did she help the old lady instead of passing on with indifference? Because of her concern for old ladies – this gives a causal difference between the fact and its contrast. Now imagine someone who has a concern for old ladies but whose habit it is to pass by old ladies who need help because she is also concerned about punctuality. On one occasion she dismisses the need to be punctual and helps the old lady: why did she stop to help the old lady instead of passing on? To give an explanation of this contrast we need a causal difference between the fact and its contrast, and her concern for old ladies will not satisfy this condition.

Consider in general what is true of an agent deliberating between two choices that are both believed by the agent to be supported by reasons, such that the agent may resolve to do either one. What explanation is there for the fact that the agent resolves to act on one set of reasons rather than the other? None of the reasons that the agent believes to support one of the choices can act as a causal difference between the fact – choosing to act on the first set of reasons and ignore the second set – and the contrast – choosing to act on the second set rather than the first, since whichever way she resolves all those reasons are present. Since we stipulated that the agent deliberates between the reasons available, no reason will serve as an explanation that is a causal difference between the resolving on one set of reasons rather than another. If this is true of our agent choosing between helping old ladies and punctuality, although it nonetheless remains true, as we saw above, the fact that she stopped instead of passing on indifferent is explained by her concern, there is no explanation in terms of motivating reasons for the fact she resolved on one concern rather than the other.

So we are left with the problematic result that our agent's resolution to act on her concern for old ladies was not a morally responsible act, and thus was not morally laudable. It looks like any account that requires such a strong connection between moral responsibility and the existence of motivating reasons that explain behaviour will have to rule much apparent moral agency as no agency at all.

THE VALUE OF KNOWLEDGE

DUNCAN PRITCHARD
UNIVERSITY OF STIRLING

0. Why care about knowledge?

One of the questions that is very rarely asked in epistemology concerns what is perhaps the most central issue for this area of philosophy. It is this: Why we should care about whether or not we have knowledge? Put another way: Is knowledge valuable and, if so, why? The importance of this question resides in the fact that it could well be that it is only if the primary focus of epistemological theorising—i.e., knowledge— is valuable that the epistemological enterprise is itself a worthwhile undertaking. The goal of this paper is to examine this issue in more detail. We will discover, perhaps surprisingly, that the value of knowledge is far from obvious.

1. The instrumental value of true belief

One way of approaching the topic of the value of knowledge is to note that one can only know what is true, and truth in one's beliefs does seem to be valuable. If truth in one's beliefs is valuable, and knowledge demands truth, then we may be at least halfway towards answering our question of why knowledge is valuable.

Truth in one's beliefs is at least minimally valuable in the sense that, all other things being equal at any rate, true beliefs are better than false ones because having true beliefs enables us to fulfil our goals. This sort of value — a value which accrues to something in virtue of some further valuable purpose that it serves — is known as instrumental value. Think, for example, of the value of a thermometer. Its value consists in the fact that it enables us to find out something of importance to us—i.e., what the temperature is.

In order to see the instrumental value of true belief, think about any subject matter that is of consequence to you, such as the time of your

crucial job interview. It is clearly preferable to have a true belief in this respect rather than a false belief, since without a true belief you'll have difficulty making this important meeting. That is, your goal of making this meeting is best served by having a true belief about when it takes place rather than a false one.

The problem, however, lies with the 'all other things being equal' clause which we put on the instrumental value of true belief. We have to impose this qualification because sometimes having a true belief could be unhelpful and actually impede one's goals, and in such cases true belief would lack instrumental value. For example, could one really summon the courage to jump a ravine and thereby get to safety if one knew (or at least truly believed) that there was a serious possibility that one would fail to reach the other side? Here, it seems, a false belief in one's abilities would be better than a true belief if the goal in question (jumping the ravine in order to save one's life) is to be achieved. So while true belief might generally be instrumentally valuable, it isn't always instrumentally valuable.

Moreover, some true beliefs are beliefs in trivial matters and in this case it isn't at all clear why we should value such beliefs at all. Imagine someone who, for no good reason, concerns herself with measuring each grain of sand on a beach, or someone who, even while being unable to operate a telephone, concerns herself with remembering every entry in a foreign phonebook. In each case, such a person would thereby gain lots of true beliefs but, crucially, one would regard such truth-gaining activity as rather pointless. After all, these true beliefs do not obviously serve any valuable purpose, and so do not seem to have any instrumental value (or, at the very least, what instrumental value these beliefs have is vanishingly small). It would, perhaps, be better—and thus of more value—to have fewer true beliefs, and possibly more false ones, if this meant that the true beliefs that one had were regarding matters of real consequence.

At most, then, we only seem able to marshal the conclusion that some true beliefs have instrumental value, not all of them. As a result, if we are to show that knowledge is valuable then we need to do more than merely note that knowledge entails truth and that true belief is instrumentally

valuable. Nevertheless, this conclusion need not be that dispiriting once we remember that while knowledge requires truth, not every instance of a true belief is an instance of knowledge. Accordingly, it could just be that those true beliefs that are clearly of instrumental value are the ones that are also instances of knowledge.

The problem with this line of thought ought to be obvious, since didn't our 'sand-measuring' agent know what the measurements of the sand were? Moreover, didn't our agent who was unable to jump the ravine because she was paralysed by fear fail to meet her goals because of what she knew? The problems that afflict the claim that all true beliefs are instrumentally valuable therefore similarly undermine the idea that all knowledge is instrumentally valuable. There is thus no easy way of defending the thesis that all knowledge must be valuable.

There is also a second problem lurking in the background here, which is that even if this project of understanding the value of knowledge in terms of the value of true belief were to be successful, it would still be problematic because it would entail that knowledge is no more valuable than mere true belief. But if that's right, then why do we value knowledge more than mere true belief?

2. The value of knowledge
So we cannot straightforwardly argue from the instrumental value of true belief that all knowledge must therefore be instrumentally valuable. That said, we can perhaps say something about the specific value of knowledge that is a little less ambitious and which simply accounts for why, in general and all other things being equal, we desire to be knowers as opposed to being agents who have mostly true beliefs but lack knowledge (or, worse, have mostly false beliefs). After all, if we want to achieve our goals in life then it would be preferable if we had knowledge which was relevant to these goals since knowledge is very useful in this respect. The idea is thus that while not all knowledge is instrumentally valuable, in general it is instrumentally valuable and, what is more, it is of greater instrumental value, typically at least, than mere true belief alone (thus explaining our intuition that knowledge is of more value than mere true belief).

Consider the following case. Suppose I want to find my way to the nearest restaurant in an unfamiliar city. Having mostly false beliefs about the locale will almost certainly lead to this goal being frustrated. If I think, for example, that all the restaurants are in the east of the city, when in fact they are in the west, then I'm going to spend a rather dispiriting evening trudging around this town without success.

True beliefs are better than false beliefs (i.e., are of more instrumental value), but are not as good as knowledge. Imagine, for instance, that you found out where the nearest restaurant was by reading a map of the town which is, unbeknownst to you, entirely fake and designed to mislead those unfamiliar with the area. Suppose further, however, that, as it happens, this map inadvertently shows you the right route to the nearest restaurant. You therefore have a true belief about where the nearest restaurant is, but you clearly lack knowledge of this fact. After all, your belief is only luckily true and that means it isn't knowledge.

Now one might think that it is neither here nor there to the value of your true belief whether it is also an instance of knowledge. So long as I find the nearest restaurant, what does it matter that I don't know where it is but merely have a true belief about where it is? The problem with mere true belief, however, is that, unlike knowledge, it is very unstable. Suppose, for example, that as you were walking to this restaurant you noticed that none of the landmarks corresponded to where they ought to be on the fake map in front you. You pass the town hall, for instance, and yet according to the map this building is on the other side of town. You'd quickly realise that the map you're using is unreliable, and in all likelihood you'd abandon your belief about where the nearest restaurant was, thereby preventing you from getting there.

In contrast, imagine that you form your belief about where the nearest restaurant is by looking at a reliable map, and thereby know where the nearest restaurant is. Since this is genuine knowledge, it would not be undermined in the way that the mere true belief was undermined, and thus you'd retain your true belief. This would mean that you would make it to the restaurant after all, and thereby achieve your goal. Having knowledge can thus be of greater instrumental value than mere true belief since having knowledge rather than mere true belief can make it

more likely that one achieves one's goals.

3. The statues of Daedalus

The previous point picks up on a famous claim made regarding knowledge by Plato. In his book, The Meno (see §§96d-100b), Plato compares knowledge to the statues of the ancient Greek sculptor Daedalus which, it is said, were so realistic that if one did not tether them to the ground they would run away. Plato's point is that mere true belief is like one of the untethered statues of Daedalus, in that one could very easily lose it. Knowledge, in contrast, is akin to a tethered statue, one that is therefore not easily lost.

The analogy to our previous discussion should be obvious. Mere true belief, like one of Daedalus's untethered statues, is more likely to be lost (i.e., run away) than knowledge, which is far more stable. Put another way, the true belief one holds when one has knowledge is far more likely to remain fast in response to changes in circumstances (e.g., new information that comes to light) than mere true belief, as we saw in the case just described of the person who finds out where the nearest restaurant is by looking at a reliable map, as opposed to one who finds out where it is by looking at a fake map.

Of course, knowledge isn't completely stable either, since one could always acquire a false, but plausible, piece of information that seems to call one's previous true information into question, but this is less likely to happen when it comes to knowledge than when it comes to true belief. In the example given earlier, suppose that the map is indeed reliable, and thus that you do know where the nearest restaurant is. Nevertheless, there might still be further misleading counterevidence that you could come across which would undermine this knowledge, such as the testimony of a friend you bump into who tells you (out of mischief) that the map is a fake. In the light of this new information, you'll probably change your belief and so fail to get to the restaurant after all.

Even so, however, the fact remains that knowledge is more stable than mere true belief. In the case just described, for example, the fact that the map had been working so far would give you good grounds to continue

trusting it, and so you might naturally be suspicious of any testimony you receive to the contrary. Suppose a perfect stranger told you that the map was a compete fake. Would that lead you to change your belief given that it has been reliable so far? Probably not. A friend's testimony carries more weight than a stranger's, but even this testimony might be ignored if you had reason to think your friend might be playing a trick on you.

If you merely had a true belief about where the nearest restaurant was, in contrast, and had no good reason in support of that true belief, then all kinds of conflicting information would undermine that belief. As we saw above, as soon as you start walking on your journey and you notice that none of the landmarks correspond to their locations on the map, then you would be liable to tear the map up in despair, even though the map is in the one respect that is important to you (how to get to the nearest restaurant) entirely reliable.

There is a good reason why knowledge is more stable than mere true belief, and this is because knowledge, unlike mere true belief, could not easily be mistaken. Imagine, for instance, a doctor who diagnoses a patient by (secretly) tossing a coin, thus leading the patient to form a particular belief about what is wrong with her. Suppose further that this diagnosis is, as it happens, correct. Clearly the doctor does not know what is wrong with the patient, even though she happened to get it right on this occasion, and neither does the patient know what is wrong with her given that she acquired her belief by listening to the doctor. The problem here is that it was just a matter of luck that the doctor chanced upon the right answer, and thus it is also a matter of luck that the patient formed a true belief about what was wrong with her. In both cases they could so easily have been wrong.

Compare this scenario, however, with that in which a doctor forms her diagnosis of the patient's illness in a diligent fashion by using the appropriate medical procedures. This doctor will (in most cases at least) end up with the same correct diagnosis as our irresponsible doctor, and thus the patient will again acquire a true belief about the nature of her condition. This time, though, the doctor and the patient will know what the correct diagnosis is. Moreover, there is no worry in this case

that this verdict could so easily have been mistaken—given that the doctor followed the correct procedures, it is in fact very unlikely that this diagnosis is wrong. Here we clearly have a case in which our goal of correctly determining the source of someone's illness is better served by the possession of knowledge rather than the possession of mere true belief because of the instability of mere true belief relative to knowledge (i.e., the fact that mere true belief, unlike knowledge, could so easily be wrong). In this sense, then, knowledge is more valuable to us than true belief alone.

For the most part, then, if one wishes to achieve one's goals it is essential that one has, at the bare minimum, true beliefs about the subject matter concerned. True belief is thus usually of instrumental value, even if it is not always of instrumental value. Ideally, however, it is better to have knowledge, since mere true belief has an instability that is not always conducive to success in one's projects. Since knowledge entails true belief, we can therefore draw two conclusions. First, that most knowledge, like most mere true belief, is of instrumental value. Second, and crucially, that knowledge is of greater instrumental value that mere true belief.

4. Is some knowledge intrinsically valuable?

At this point we might wonder whether the value of knowledge is only ever instrumental. That is, we might wonder whether the value of knowledge is always dependent upon what further goods, such as gaining relief from your illness, which knowledge (in this case of the correct diagnosis of your illness) can help you attain. Intuitively, this claim is too strong in that there do seem to be certain kinds of knowledge which have a value which is not purely instrumental. Put another way, some kinds of knowledge seem to have an intrinsic value.

If something has intrinsic value, then it is valuable in itself, regardless of what, for instance, it enables one to do. Friendship is intrinsically valuable, for example. We don't value our friends because they are useful to us (though having friends is undoubtedly useful), but simply because they are our friends. If you valued someone just for what they can do for you (help you to make more money, for example), then you wouldn't count as their friend. Put another way, although there is clearly

an instrumental value to having friends—they improve our quality of life, for example—the true value of friendship is not instrumental at all, but intrinsic to the friendship itself.

In order to see how knowledge could be intrinsically valuable, think of those types of knowledge which are very refined, such as wisdom— the sort of knowledge that wise people have. Wisdom is clearly at least instrumentally valuable since it can enable one to lead a productive and fulfilled life. Crucially, however, it seems that knowledge of this sort would still be valuable even if, as it happens, it didn't lead to a life that was good in this way. Suppose, for instance, that nature conspires against you at every turn so that, like the Biblical character Job, you are subject to just about every dismal fate that can befall a person. In such a case your knowledge of most matters may well have no instrumental value at all because your goals will be frustrated by forces beyond your control regardless of what you know.

Nevertheless, it would surely be preferable to confront such misfortune as a wise person, and not because such wisdom would necessarily make you feel any better or enable you to avoid these disasters (whether wise or not, your life is still wretched). Instead, it seems, being wise is just a good thing, regardless of what further goods it might lead to. That is, it is something that is good in itself; something which has intrinsic value. And notice that this claim marks a further difference between knowledge and mere true belief, since it is hard to see how mere true belief could ever be of intrinsic value.

There may be stronger claims that we can make about the value of knowledge, but the minimal claims advanced here suffice to make the study of knowledge important. Recall that we have seen that knowledge is at least for the most part instrumentally valuable in that it enables us to achieve our goals, and that it is more instrumentally valuable in this respect than true belief alone. Moreover, we have also noted that some varieties of knowledge, such as wisdom, seem to be intrinsically valuable. Clearly, then, knowledge is something that we should care about, and given that this is so it is incumbent upon us as philosophers to be able to say more about what knowledge is and the various ways in which we might acquire it. These are key goals of epistemology. [1]

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Notes

1. This paper draws on material published in Pritchard (2006, chapter 2). The classic study of the value of knowledge in the recent literature is Kvanvig (2003). For a survey of the recent literature on the value of knowledge, see Pritchard (forthcoming).

What mathematical knowledge could not be

PHILIP A. EBERT

Introduction

This survey paper will critically discuss four different strategies to explain our knowledge of mathematics. In the first section I will outline Benacerraf 's dilemma as put forth in Benacerraf 's famous paper "Mathematical Truth"1 –

a dilemma faced by any account of mathematical knowledge. The aim of this section is to clarify and discuss the semantic and epistemic constraints that Benacerraf (explicitly and implicitly) imposes, and show how they give rise to his well-known dilemma. In the second section I will review four strategies to overcome this dilemma as they occur in the philosophical literature. The first two platonistic strategies comply with the semantic constraint but, I will argue, provide insufficient answers to the epistemic constraint, while the other two, nominalistic strategies either reject the idea of mathematical knowledge altogether or fail the semantic constraint. In the last section, I will elicit, on the basis of my discussion of the four conceptions, what I label the fundamental assumption. I will argue that it is presupposed by all four strategies and suggest that a rejection of this assumption will give rise to a different type of platonistic response. A thorough discussion of this fifth alternative will, however, be postponed to another occasion.

1 Benacerraf 's dilemma

In his seminal paper "Mathematical Truth"2 Benacerraf outlines a dilemma that every account within the philosophy of mathematics faces. The dilemma arises from the need for any such conception to satisfy two constraints whose mutual resolution, however, seems impossible. The first constraint concerns the semantic theory adopted for our mathematical discourse. Here the demand is to have a "homogenous semantical theory in which the semantics for the statements parallel the semantics for the rest of the language." (p.403)

The second constraint is epistemological in nature and demands that "the account of mathematical truth mesh with a reasonable epistemology." (p.403) Benacerraf 's dilemma, in its simplest form, arises when one attempts to conform to those two constraints by appeal to the prevalent views in semantics and epistemology: The standard view to comply with the first constraint – a Tarskian theory of truth – imposes an ontology which is incompatible with the standard view in epistemology, which, at the time of Benacerraf 's publication3 , comprised causal constraints on knowledge of any type of ob ject. In order to explain the dilemma and the resulting challenge more clearly, I will have a closer look at the two main constraints which each involve two components.

> "Another way of putting this first requirement is to demand that any theory of mathematical truth be in conformity with a general theory of truth – a theory of truth theories, if you like, – which certifies that the property of sentences that the account calls "truth" is indeed truth. This, it seems to me, can be done only on the basis of some *general theory for at least the language as a whole*." (p. 408, my italics)

The only candidate to satisfy this demand is, according to Benacerraf, a Tarskian account of truth, where truth is spelled out by appeal to reference and satisfaction. Here, the basic idea is to assign semantic values to the different semantic components of a sentence and to then correlate the truth or falsity of a sentence with those semantic values.6

The second requirement of the semantic constraint adopted by Benacerraf is to take the surface grammar of sentences of the discourse at face value. Consider his two examples:

1. There are at least three large cities older than New York.

2. There are at least three perfect numbers greater than 17.

If we take the surface grammar at face value we should regard both sentences as having the same structure, namely:

Benacerraf reinforces this second requirement when he writes that "we should not be satisfied with an account that fails to treat (1) and (2) in parallel fashion, on the model of (3)." (p. 408) Hence, in addition to adopting a Tarskian – referential – theory of truth, the semantic constraint also involves the demand to respect the surface grammar and hence to take it without further qualification that what seem to be singular terms, such as "New York" and "17", are singular terms.

To sum up, the semantic constraint demands a general and systematic theory of truth to be applied to the mathematical language; and it also demands that we respect the surface-grammar of the mathematical discourse in so applying our theory of truth. Consequently, the standard view which satisfies these demands does interpret mathematics by analogy with empirical sentences. Both are analysed using predicates, quantifiers and singular terms and satisfaction-conditions for truth. Crucially, in the case of mathematics, number-terms are regarded as singular terms and as such are required to denote objects. In addition, the objects that number-terms refer to are intuitively regarded as non-spatio-temporally located and are thus abstract. The resulting position – the standard view – is one which genuinely deserves the label 'platonistic', as Benacerraf notes.

The merit of the standard view for the semantic constraint is, according to Benacerraf, that the semantics of mathematics nicely meshes with that of other discourses. Consequently, mixed discourses – where mathematical and empirical terms are used in the same context – pose no additional problem. Further to these virtues of parsimony and simplicity, Benacerraf notes that a Tarskian theory of truth "is the only viable systematic general account we have of truth" in order to account for the first requirement. Finally, note that taking the surface-grammar at face value keeps the truth-conditions of mathematical discourse in line with the thoughts intuitively expressed by such sentences. Hence, respecting the surface-grammar guarantees that what seems to be the subject of our mathematical discourse is its subject and thus what seems to be known and thought about by the subject is what is known and thought about by the subject.

Let us now turn to the second epistemological constraint proposed by

Benacerraf. At first sight, it merely comprises a minimal assumption that hardly seems controversial: we have mathematical knowledge, which "is no less knowledge for being mathematical" (p.409). Subsequently, the demand of the second constraint is just that the account of mathematical truth be compatible with an account of knowledge that renders such truths knowable.

According to Benacerraf, the standard way of complying with this second constraint, and the only viable general account of knowledge, is a causal account. The idea, roughly, is that for a sub ject X to know that p, there has to obtain some type of causal relation between the sub ject X and the ob jects, or other items, involved in the sub ject matter of p. This account can be easily motivated by appeal to our knowledge of medium-sized, everyday ob jects, which does seem to involve causal relations.

However, adopting the standard view – a Tarskian theory of truth – to comply with the semantic constraint and adopting the standard way – causal theory of knowledge – to comply with the epistemological constraint seems to lead to an impasse, and brings Benacerraf's dilemma to the fore: If the truth-conditions of mathematical statements are given by the standard semantical account, then an arithmetical statement involving number terms should be regarded as making reference to numbers as ob jects that exist in an abstract realm. However, these ob jects seem unaccountable within the standard view in epistemology, as clearly no causal connection between an object of this sort and the presumably knowing sub ject can be made out. Thus, no explanation of how we can come to know that the truth-conditions of a mathematical statement obtain can be offered. Crucially, note that what Benacerraf outlines is a genuine dilemma in that either of the two constraints (or even both) can be relaxed – either of the two standard views can be dismissed and a new theory of truth or new epistemology can be put into its place. It is not a direct attack (as often thought to be) on the platonistic view of semantics.

Before I outline various general strategies for resolving the dilemma, I will briefly consider a misled interpretation of the epistemological constraint. This will lead to a strengthening of the simple version of the dilemma.

It would be an inadequate platonist response to the dilemma to merely reject the causal conception of knowledge in the light of recent criticisms. Certainly, the causal conception of knowledge does not always seem adequate for medium sized ob jects which provided its initial motivation, nor is it prima facie compatible with knowledge of the future or even facts that can be construed as involving arguably less suspect abstract ob jects such as the University of St Andrews, Dundee United F.C. or Apple Computers. Thus, so the platonist could argue, the causal theory of knowledge should not be adopted, since it clearly fails to be a simple and general account of knowledge that would qualify as the "only viable systematic general account we have of" knowledge. Consequently, Benacerraf 's dilemma would vanish in its initial form since there are no fully general causal constraints on knowledge.

Although I am in general sympathetic to the idea that the causal theory of knowledge is insufficient as an account of our knowledge in general, this response, however, misses the crucial point of Benacerraf 's dilemma. The dilemma, in its strongest form, not only points to an incompatibility between two standard views within two areas in the philosophy of mathematics: it also aims to highlight an integration problem in the philosophy of mathematics.7 Accordingly, the epistemological constraint does not necessarily depend on adopting a causal account of knowledge. Rather, what is needed is an epistemological account that is able to integrate mathematical truths and thereby provide an explanation of how it is possible that mathematical truths, whose truth-conditions are spelled out using platonistic ontology (according to the the standard view of the semantic constraint), can be known to obtain by the sub ject.8 Rejecting the causal account does little to explain how such truth-conditions can be reliably known to obtain.

It is this stronger version of the Benacerraf dilemma – in the form of an integration problem – which will be the focus of the following sections. But, for clarity, I believe we should distinguish two requirements towards providing a fully integrated account. An epistemology for mathematics – which involves abstract objects – not only needs to explain how we know that the truth-conditions of a mathematical statement obtain, but, in addition, it also needs to explain how we can have (directed) beliefs

about the abstract ob jects that make the mathematical statements true. To put the point differently, it needs to explain how in thought we can have access or refer to ob jects that are not spatio-temporal.9
This idea can be further explicated by considering the following passage of Hartry Field's interpretation of the dilemma,10 which also dispenses with the purely causal constraint. He writes:

> "Benacerraf 's challenge – or at least, the challenge which his paper suggests to me – is to provide an account of the mechanisms that explain how our beliefs about these remote entities can so well reflect the facts about them." ((Field, 1980), p. 26)11

Here, it seems to me, two issues are at hand in the challenge. The first issue is to explain how, in principle, we can successfully talk or have beliefs about abstract ob jects in the first place. The second issue is to explain, more specifically, how we are justified in thinking that those mathematical beliefs are true, or, as Field would put it, how those beliefs reliably track the mathematical facts.

In conclusion to these considerations, I think we can now see that there are four separate requirements (the first two involved in the semantic constraint and the other two in the epistemological constraint) that collectively make up Benacerraf 's dilemma and that collectively need to be addressed:

1. Homogeneous semantic theory
The demand that we adopt a general and systematic theory of truth, which – for Benacerraf – should be a Tarskian Theory of truth.
2. Surface-grammar
The demand to respect the surface grammar of mathematical discourse.
3. Reference and object-directed thought
The demand to explain how the ob jects posited by the semantic theory can, in principle, be in the range of directed thought and talk of the sub jects.
4. Knowledge
The 'integration challenge': The demand to reconcile the truths of the sub ject matter with what can be known by ordinary human thinkers.

Crucial here is to provide an explanation of how a sub ject can have mathematical knowledge and on what basis the sub ject can claim such knowledge.

2 Strategies to resolve Benacerraf 's dilemma

In this section I will review four different strategies to resolve Benacerraf's dilemma. I will begin with a version of platonism which regards mathematical knowledge as a special kind of knowledge that has its own special source and so is distinct in kind from knowledge of other sub ject matters.

2.1 *Intuitive* platonism

Intuitive platonism adopts the standard view of the semantic constraint, conceiving of the mathematical language as referring to self-subsistent, abstract mathematical entities, and also respects the surface-grammar of the mathematical discourse. Our knowledge of such 'remote' entities is explained by the fact that in the case of mathematics we are concerned with a special type of knowledge, which in the relevant respect is basic. The idea here is to break with the demand that a general ly applicable account of knowledge is needed for every discourse. Instead, the axioms of mathematics and the rules of inference – from which the theorems of mathematics are derived – are regarded as basic in the sense that they cannot be inferred from, and so be known in virtue of, even more fundamental principles. Rather, a subject's non-inferential knowledge of the axioms and the rules of inference has its source in the special faculty of intuition, which, similar to the faculty of perception, provides direct knowledge of the truth of the basic axioms.

The main proponents of intuitive platonism within the philosophy of mathematics have been Kurt Godel and more recently Charles Parsons[12]. To characterise this type of platonism more precisely, the following often-cited quotation from Godel highlights the role intuition is supposed to play here:

> "But despite their remoteness from sense-experience, we do have something like a perception of the ob jects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have less confidence

> in this kind of perception, i.e. mathematical intuition, than in
> sense-perception." (G ̈odel, 1947), p.483-4

This version of platonism, however, faces various difficulties in providing a satisfactory answer to the two epistemic requirements in Benacerraf 's dilemma. For one thing, note the transition in this quotation from knowledge of objects ("the ob jects of set theory") with which mathematical intuition is concerned to knowledge of the truths of axioms ("the axioms force them selves on us") which G ̈odel aims to underwrite by the faculty of intuition. Leaving aside what underwrites this transition, it remains unclear what the mark of a successful intuitive grasp of such an abstract ob ject is. Just claiming that we can perceive these ob jects and thereby regard the mathematical axioms as intuitively compelling, obvious, or as somehow "forcing themselves upon us", seems insufficient as a genuine justification of our cognitive beliefs in the truth of the mathematical axioms.

To highlight this further, consider in analogy the scenario in which we postulate a faculty of "perceiving other minds" which provides immediate knowledge of other minds and thereby accounts for the obviousness or intuitive compellingness of certain beliefs about them. No-one (in their right mind) would regard this as a sufficient explanation and justification of our beliefs about other minds.

Also, and connected to this weakness, intuitive platonism has to be able to account for the fal libility of this faculty. It would be insufficient to say that Frege's faculty of intuition did let him down when he postulated Basic Law V as an axiom, without explaining why it did and why it does not in other cases where the target statements are consistent.13

Various additional concerns could be raised but what seems to be at the heart of most criticisms14 is that a postulated faculty of intuition fails to provide a genuine explanation of our access to and knowledge of abstract entities, since it is just built into the faculty of intuition – as a brute fact – that it does enable such access and knowledge. Hence, crudely put, this type of platonism seems more like an acknowledgement of the inability to provide an explanation of our knowledge of mathematics than a genuine solution.15

In addition, the intuitive platonist also runs the risk of dislocating our mathematical knowledge from everyday and, more crucially, scientific knowledge. How on this view is mathematical knowledge embedded and interactive within the scientific corpus of knowledge? It is this version of the well-known application problem, which concerns how mathematical knowledge is applicable in empirical science, that proves especially challenging for the intuitive platonist. And exactly at this point is where a new type of platonism can be located, one which rejects both the view that mathematics has it's own epistemology, and that it gives rise to a special sort of a priori knowledge.

2.2 *Naturalised* platonism

Naturalised platonism, whose principal author is Quine16 , regards mathematical knowledge as being on a par with scientific knowledge. So, mathematical knowledge is part of our theoretical knowledge, just like knowledge of physics or chemistry, and the ob jects of mathematics are theoretical objects just like electrons, neutrinos or strings are theoretical ob jects posited by the physical theories. Therefore, there is neither the need for a special faculty of intuition to explain mathematical knowledge nor does mathematical knowledge enjoy a special status – as a type of a priori knowledge.

But, crucially, then how does the naturalised platonist explain mathe-matical knowledge, even if it is merely theoretical? After all, this version of platonism also adopts a Tarskian semantic theory to comply with the semantic constraint of Benacerraf 's dilemma and, so, regards the objects of mathematics as abstract ob jects.

This integration challenge has received an answer by the naturalised platonist in the form of the now well-known Quine-Putnam indispensability argument.17 The argument can be presented as follows:

Premise 1: Mathematics is indispensable to our scientific theories, in that they can neither be formulated nor practised without mathematical vocabulary and inferences.

Premise 2: If mathematics is indispensable to our accepted scientific theories, then if those scientific theories are true then the mathematics involved in scientific theorising is true.
Intermediate Conclusion 1: If scientific theories are true then the mathematics involved in scientific theorising is true.

Premise 3: Scientific theories are true.

Intermediate Conclusion 2: The mathematics involved in scientific theorising is true.

Premise 4: If mathematics is true, then there are the abstract entities to which it purportedly refers, such as numbers, functions, sets.

Conclusion: Abstract entities, such as numbers, functions and sets that are appealed to in mathematical theories which are involved in scientific theorising, exist.

It is in virtue of these pragmatic considerations that the naturalised platonist aims to incorporate his platonist conception of mathematics within a naturalised epistemology, whereby all knowledge is merely empirical. Obviously crucial here is that mathematics actually is indispensable to science in the relevant respect – a claim that has been challenged by Field, and which will be discussed in the next section. But even granting that mathematics is indispensable, two issues remain: Firstly, is a conception for which every statement is empirical and as such "up for revision" stable and, secondly, if it is stable, how exactly do these pragmatic considerations resolve the two epistemic challenges?

The first question has received much discussion in recent years. (Wright, 1986) argues that a Quinean position – which is a form of global empiricism – is intrinsically incoherent. The argument itself is very intricate and I don't propose to discuss it here. The second question, however, is more pertinent to the current discussion and concerns the adequacy of the naturalised platonist answers to the epistemological issues about our knowledge of mathematical ob jects.

Just like the intuitive platonist, the naturalised platonist does not pro-

vide much in terms of an explanation of our access to, or knowledge of, abstract ob jects. The indispensability argument might at best provide an argument that we are justified in thinking that there are abstract objects conceived of as theoretical entities. However, note that on this perspective a plausible element of mathematical thinking is lost. As Frege noted, "in arithmetic we are concerned with ob jects that we come to know not as something alien, from without through the medium of the sense, rather they are directly posited to reason, which, as its nearest kin, it can completely grasp." ((Frege, 1884), §105 my translation) Hence, the "charm" of mathematics as "the reason's proper study" is lost from a naturalised platonist perspective, as knowledge of mathematics and knowledge of the ob jects of mathematics is merely justified indirectly by its involvement in scientific theories that are true.

Moreover, it is worth noting that knowledge of mathematics is parasitic upon (global) scientific realism (premise three) which is needed to arrive at the conclusion that mathematics is true. And, even granting the soundness of the indispensability argument, only a small part of mathematics will be justified by its application in science and thus the line between applied and pure mathematics becomes of crucial epistemological significance, since only the former, and not the latter (provided the above indispensability argument is all we have), can be justifiably regarded as true.18

Thus, one misgiving about naturalised platonism is that, even granting for the moment that the indispensability explains how a sub ject can have knowledge of parts of mathematics, this position is failing to address Benacerraf 's dilemma in full generality. The challenge is how mathematics in general can be integrated into a thinker's corpus of knowledge, and not how some parts can be so integrated. So, I think a position that aims to explain all of mathematics and so tackles Benacerraf 's dilemma in full generality is what is needed and desirable. Also, I find it hard to regard the indispensability of mathematics as an adequate explanation of our knowledge of mathematical entities and mathematical statements. Appealing to the need for mathematics in a presumably truth-apt scientific discourse does not – so at least it seems to me – provide the right type explanation of a thinker's access to and knowledge of mathematical entities. Lastly and rather worryingly, the

very idea that mathematics is indispensable to science is challengeable (and we shall review this challenge to naturalised platonism in the next section). Therefore, I think that these misgivings raise serious doubts as to whether the naturalised platonist position provides an adequate response to Benacerraf's dilemma.

To summarise, both intuitive and naturalised platonist positions suffer from an inability to provide satisfactory answers to the epistemic issues of Benacerraf's dilemma. Both adopt the standard view to comply with the semantic constraint, yet they fail to account for the epistemological challenges. These failures can be regarded as motivating two alternative positions next to be discussed. The first, error-theoretic nominalism, aims to avoid the problematic ontology, without thereby rejecting the standard view in semantics, by claiming that mathematics, taken at face-value, is actually false. This conception is – in some respects – in the tradition of naturalised platonism in that it regards all knowledge as theoretical, if it exists at all, but rejects the indispensability of mathematics to science, thereby leaving no possible theoretical reason to accept mathematical entities in the first place. The second conception I shall call reconstructive nominalism. It also accepts a Tarskian account, but regards the surface-grammar of the mathematical discourse as misleading. Mathematical ontology, accordingly, is not what it seems to be.

2.3 *Error-theoretic* nominalism

This strategy is famously proposed and explored by Hartry Field in several of his writings.19 It can be motivated as a reply to the naturalised pla tonist who, according to the nominalist, has not gone far enough. What motivated naturalised platonism was the idea that we only have theoretical, i.e. empirical knowledge and thus no additional faculty has to be appealed to in order to account for mathematical knowledge. The error-theoretic nominalist adopts the main feature of naturalised platonism – that there is only empirical knowledge – while additionally avoiding any commitments to abstract entities whose knowledge is hard to explain.

As the naturalised platonist conception was characterised above, the only reason it provides for thinking that there are numbers, sets, etc is the previously outlined indispensability argument. Field regards this a

valid argument but denies its soundness by rejecting the first premise – the claim that mathematics is indispensable to science. Crucially, Field works with an understanding of the underlying notion of indispensability which has to do not merely with the expressive resources gained by using mathematics, but also with the fact that mathematics is essential in establishing (proving) theorems and making predictions. Hence, to undermine the indispensability of current mathematical theory, it needs to be shown that there is an alternative theory that does equally well in establishing theorems and making predictions, but which does not involve commitments to numbers, sets, etc.

Field attempts exactly this and provides a framework that, according to him, does equally well, but that makes (arguably) no reference to abstract objects.20 Consequently, Field rejects the indispensability argument and with it he rejects what the regards as the only good motivation to believe in abstract ob jects. Hence, he adopts an error-theory for standard mathematics in that, taken at face-value, mathematics is false since it has ontological commitments to things that we have no reason to believe exist.21 Nevertheless, he thinks that we are still entitled to use a false theory and that it is desirable to do so since it is simpler and helps to "speed up inferences" so long as it is conservative. The relevant notion of conservativeness is the following:

Field's notion of conservativeness
"A mathematical theory M is conservative iff for any assertion
A about the physical world and any body N of such assertions, A
doesn't follow from N + M, unless it follows from N alone."((Field,
1982), p. 58)

The idea is that if mathematics is conservative, it is acceptable to use mathematics since it won't lead to any conclusions that could not be arrived at without mathematics. So, mathematics can be used to "speed-up" inferences or, in general, to make life easier for scientists without the need to endorse its truth and thereby its ontological commitments. Consequently, the basic notion for the error-theoretic nominalist is conservativeness and his credo is that "a mathematical theory must be conservative but need not be true".22

As can be expected of such a radical view, there is an extensive literature on Field's approach23 that I won't attempt to survey here. Rather I will assess how such an account would resolve Benacerraf 's dilemma, assuming the more specific criticisms are resolved.

In some respects, Field's resolution of the dilemma is simple but radical: We are misled in thinking that mathematics is true – quite the opposite, it is false. Still it is valuable because it is conservative (or at least parts of it are). Just like the two types of platonism above, the error-theoretic nominalist adopts the standard view to address the semantic constraint. He adopts a Tarskian theory of truth and he respects the surface grammar – but denies the truth of the mathematical discourse, while granting its usefulness cashed out in terms of conservativeness. So, provided error-theoretic nominalism can overcome various technical difficulties in order to account for enough science, consistent with maintaining that mathematics is false but conservative, this position aims to overcome Benacerraf 's dilemma by a radical route: denying mathematical knowledge altogether. Moreover, there is no need to account for a thinker's reference to and ob ject-directed thought about mathematical entities.

However, is this a promising strategy to resolve Benacerraf 's dilemma? I don't think so. A satisfying solution to the dilemma should not consist in giving up the basic assumption that we have mathematical knowledge. In a similar vein to my criticism to naturalised platonism, I think what is needed is a direct solution – taking Benacerraf 's challenge head on – and integrate mathematical knowledge in general. Error-theoretic nominalism, in contrast, merely acknowledges defeat by dropping Benacerraf 's main assumption that mathematical knowledge "is no less knowledge for being mathematical" (p.409). In turn, it seeks to explain why, in spite of there being no mathematical knowledge, we can pursue mathematics without a bad conscience. So, I think, in the context of a resolution to Benacerraf 's dilemma, Field's approach is a mere last resort. I will therefore, leave aside error-theoretic nominalism and continue to explore the possibility for a more suitable resolution of Benacerraf 's dilemma. Reconstructive nominalism is an alternative version of nominalism that shares the general scruples about abstract ob jects, but, less radically than Field, aims to retain the truth of mathematics, even if

it is not truth in virtue of the properties of numbers, sets, functions, etc. This strategy will be discussed in the next section.

2.4 *Reconstructive* nominalism

The reconstructive nominalist view can also be regarded as a response to naturalised platonism. Yet, it adopts a different strategy to avoid the problematic ontology, which seems to pose a serious problem for our knowledge of mathematics and so for a satisfying solution to Benacerraf's dilemma. In contrast to error-theoretic nominalism that rejects the problematic ontology and the indispensability of mathematics, the reconstructive nominalist trades ontology for ideology. To explain, he rejects the following crucial move in the indispensability argument:

Intermediate Conclusion 2: The mathematics involved in scientific the-orising is true.
Premise 4: If Mathematics is true, then there are the abstract entities to which it purportedly refers, such as numbers, functions, sets.
Conclusion: Abstract entities, such as numbers, functions and sets that are appealed to in mathematical theories which are involved in scientific theorising, exist.

The reconstructive nominalist challenges premise four that the truth of mathematics commits one to the objects purported to be referred to by the mathematical terms. And to underwrite the motivation for rejecting this conditional, the reconstructive nominalist discards the second requirement of the semantic constraint, which involves the demand to take the surface-grammar of the mathematical discourse at face-value.

Rejecting this assumption opens up the possibility of reconstructing mathematical discourse in various ways. One proponent of this strategy is Hellmann who adopts a version of Modal Structuralism 24 . The idea is to trade mathematical ontology – reference to numbers, sets, etc – for added ideology, namely the use of modal discourse. Mathematics is now conceived as concering possible structures (and objects).

This conception is indeed a type of nominalism, as it refrains from reference to and quantification over existing abstract objects and instead merely commits one to possible entities. In the case of arithmetic, the

commitment is to a possible w-structure – a structure that exhibits the properties normally assigned to numbers and so makes the Peano axioms true.25 Various formal details need to be attended to, to make this reconstructive strategy work.26 Here, however, I will leave these formal issues aside and, just as above, note various difficulties with this approach and assess how it fares with regard to overcoming Benacerra's dilemma.

Reconstructive nominalism typically adopts Tarski's theory of truth but denies the need to respect the surface grammar. Hence, such a conception denies the basic presumption that what seems to be referred to, or what seems to be thought about when doing mathematics, is what is referred to or thought about. According to reconstructive nominalism, there are no such ob jects as numbers underlying our mathematical thought and talk. We are systematically misguided in thinking that the surface grammar represents reality. To some, this might seem like a hard bullet to bite and, in general, I believe that an account which does respect the surface-grammar has advantages over a reconstructive account.

The important issue, however, is the epistemic constraint. The basic idea is that by avoiding the critical ontology (abstract ob jects) by appeal to possible structures, this type of reconstructive nominalism can overcome Benacerraf 's dilemma. So, the thought had better be that knowledge of merely possible structures is easier explained and justified than knowledge of actual abstract ob jects. I think, however, that exactly this claim can be challenged.27 First, to clarify the modal structuralist position, it should be noted that the possible structures are structures of ob jects. For there to possibly be an w-structure to make the Peano axioms true, there have to be possibly infinitely many ob jects. Otherwise structures per se seem every bit as abstract as numbers, sets, etc. As a nominalist, Hellman should not be committed to the possible existence of infinitely many abstract ob jects because it would follow that it is contingent whether there are abstract objects – a curious contingency that needs to be explained.28 However, Hellman is aware of these difficulties and commits himself to the possibility of a concrete w-structure, i.e. that there could be infinitely many concrete ob jects (making up an w-structure).

But, then how can one know this modal claim? What explains our knowledge, if we have it, that there could be infinitely many concrete objects? One option is to argue from the mere conceivability of there being infinitely many concrete ob jects that it is possible that there are – but the extent to which conceivability is a good guide to possibility is a further issue. In general we can say that, just as in the case of abstract ob jects, a variant of Benacerraf 's dilemma can be raised for the modal realm and the possible existence of certain ob jects.29 Unless it is clear that the latter type of knowledge – modal knowledge – is easier to explain and that the general scruples about abstract ob jects should be upheld, there are no advantages (but merely disadvantages based on added complexity and the rejection of surface-grammar) to such a reconstructive approach.30

2.5 The fundamental assumption of the four strategies

We have seen that apart from intuitive platonism, the other three strategies either truncate the knowledgeable part of mathematics, deny any mathematical knowledge, or turn mathematical knowledge into modal knowledge.

Hence, these three strategies have not taken the dilemma head on; rather, they give up on basic components that gives rise to the dilemma. Considering that neither these indirect responses nor the one direct response (intuitive platonism) offer a satisfying account to resolve Benacerraf epistemic constraints, I here want to explore whether there is an assumption that is shared by these four strategies. By identifying and then challenging it, we might be able to arrive at an alternative conception that holds the key to resolving Benacerraf 's challenge. I think, that each strategy is committed to the following conditional, which I will label the fundamental assumption :

> If there is a priori mathematical knowledge and the mathematical discourse is construed at face-value, then there has to be some form of acquaintance with the ob jects involved that underwrites this knowledge.

This assumption can reasonably explain why the intuitive platonist postulates a "perception-like" faculty – the faculty of intuition – which provides a form of acquaintance with the abstract ob jects. Having engaged in this (mysterious) interaction with the abstract ob jects

results, according to intuitive platonism, in a priori knowledge of the abstract ob jects and the relevant axioms purported to be about these ob jects. Naturalised platonism also accepts this conditional but, in contrast to intuitive platonism, denies that there can be a perception-like mathematical faculty or any other form of acquaintance with abstract ob jects, and accordingly denies that there can be any type of a priori knowledge of mathematics. As a result, there is a need to resort to broadly empirical and scientific considerations to explain mathematical knowledge. Both error-theoretic and reconstructive nominalism challenge the sufficiency of a broadly empirical epistemology of mathematics and both nominalist positions can also be interpreted as adopting the fundamental assumption. Error-theoretic nominalism adopts an even stronger assumption, namely that any type of knowledge has to involve some form of interaction or acquaintance, however indirect. Since there can be no such interaction with mathematical entities there is no mathematical knowledge in general. Reconstructive nominalism in contrast also accepts the truth of the fundamental assumption but then denies the component of the antecedent which claims that mathematics is to be construed at face-value.

Reflecting on this assumption offers the opportunity to rethink the platonist strategy. What exactly would be involved in rejecting the truth of the fundamental assumption? If it were rejected, it should be possible to have a priori knowledge of mathematics and the mathematical ob jects without having an acquaintance with the ob jects that mathematics is about. So, there is no need to account for an initial interaction with the sub ject-matter in order to found knowledge of that subject-matter.

Thus a challenge of this subject-matter first idea would comprise the claim that a sub ject can have and justifiably claim knowledge of numbers without direct acquaintance as intuitive platonism demands and without resorting to purely pragmatic considerations which threaten a genuine explanation of mathematical knowledge. Instead, so the suggestion might go, we can have knowledge of numbers by reflecting upon statements about the ob jects in question – that is we can, in some sense, gain knowledge by linguistic competence and by mastering of the mathematical discourse. I think pursuing this new (fifth) strategy is definitely worth a try31 and some philosophers have

attempted to take this linguistic turn for our knowledge of mathematics, yet a proper discussion of this strategy will have to be postponed to another occasion.32

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Notes
1. (Benacerraf, 1973).
2. See (Benacerraf, 1973) all unspecified references in this paper will be to the paper as published in (Benacerraf and Putnam, 1983).
3. See for example (Goldman, 1967) or (Harman, 1973) as mentioned by Benacerraf.
4. See especially (Hale and Wright, 2002) who distinguish these two issues. I draw on parts of their discussion of Benacerraf 's dilemma.
5. So a similar "Benacerraf 's dilemma" can also be posed for ethical and modal discourse.
6. Much more can be said about a Tarskian theory of truth, but the details are not important here.
7. This terminology was first introduced by (Peacocke, 1999) who himself refers to Benacerraf 's dilemma as a prototype of the integration problem.
8. Note that a solution to the integration problem need not start with a semantic theory as I do here. It is open to start with an epistemology and then aim to integrate a semantic theory within it.
9 The question of what our mathematical beliefs are about is underlying Benacerraf 's earlier discussion in (Benacerraf, 1965).
10 See (Field, 1980) especially the introduction to this book, p. 20-30

11. Dialectically there is a difference between Field's presentation and mine. He continues the above quote in the following way: "The idea is that if it appears in principle impossible to explain this, then that tends to undermine the belief in mathematical entities, despite whatever reason we might have for believing in them." (op.cit.) I don't think that this is how Bencacerraf 's dilemma should be understood. I believe it is a genuine dilemma while Field regards it more or less as a challenge to the platonist.

12. For example in (Parsons, 1979).

13. Frege in his Grundgesetze der Arithmetik (Frege, 1903) put forth Basic Law V and regarded it as a self-evident (logical) truth. Bertrand Russell showed in 1901 in a famous letter to Frege that his axiom is inconsistent.

14. See for example (Hale and Wright, 2002) for extensive criticism of Parson's and Godel's views.

15. There is however a recent resurrection of the idea of a faculty of intuition in current epistemology in the works of BonJour in his (BonJour, 1998) and (Sosa, 2005), which I won't be able to cover here.

16. See his (Quine, 1986) but there are various others who hold similar positions, such as (Resnik, 1997) and (Shapiro, 1997).

17. Locus classicus is (Putnam, 1971). For an extensive discussion and recent defence of the indispensability argument, see (Colyvan, 2001).

18. There are further issues, for example how and if at all classical logic could be justified in virtue of application in science. See for example (Shapiro, 2005) for such discussion.

19. See his (Field, 1989) and (Field, 1980). Chapter 2 of (Field, 1982) provides a non-technical introduction.

20. Field is able to provide a theory without reference to mathematical ob jects that is able to capture Newtonian physics. Field's theory, however, makes reference to space-time points that, some have argued, are best regarded as abstract entities. I won't pursue this line of worry here.

21. It can be challenged that this consideration suffices to establish an error-theory about mathematics. It seems rather that agnosticism should follow from this consideration. I will leave further discussion of this issue aside and grant that an error-theory about mathematics can be motivated in this way.

22. (Field, 1982), p. 58.

23. A chronological collection of the most important literature is:

(Malamet, 1982), (Shapiro, 1983), (Shapiro, 1984), (Hale, 1987), as well as most of the papers in the collection (Irvine, 1990). For a very detailed survey of Field and his critics consult (MacBride, 1999) and for a detailed account and criticism of the technical framework see (Urquhart, 1990), as well as (Burgess and Rosen, 1997) who also provide a very nice reconstruction of the nominalism-platonism debate. For some replies (especially to Shapiro) see (Field, 1985).

24. (Hellman, 1989)

25. On one reading of nominalism, namely Goodman's and the early Quine's, this approach would not be considered nominalistically acceptable. Compare: "Goodman and I got what we could get in the way of mathematics on the basis of a nominalist ontology and without assuming an infinite universe. We could not get enough to satisfy us. But we would not for a moment have considered enlisting the aid of modalities. The cure would have been far worse than the disease." (reply to Charles Parson in (Hahn, 1986) as quoted from (Burgess and Rosen, 1997), p. 248. I won't here enter the dispute what is distinctive of nominalism, and continue to regard Hellman as a nominalist.

26. For example providing the right type of translation from standard mathematics to modal statements, the problem of trivial conditionals, etc. For details see (Hellman, 1989) and (Burgess and Rosen, 1997).

27. See especially (Hale, 1996).

28. This is a point made in Hale (Hale, 1996). Consult this paper for further discussion and explanation.

29. See for an illuminating discussion (Stalnaker, 1996).

30. I have have to forgo discussion of other structuralist views, such as those of (Shapiro, 1997) and (Resnik, 1997), and the more pessimistic position which claims that there is no, or not just one solution, to the Benacerraf challenge, defended in (Azzouni, 1994) and (Balaguer, 1998) respectively.

31. After all, it resulted in a PhD for the author of this paper.

32. Frege in his Grund lagen der Arithmetik ((Frege, 1884)) has often been identified as the first proponent of this fifth strategy. The so-called Neo-Fregeans are defending a view that is broadly in line with the fifth strategy. See especially (Wright, 1983), (Hale, 1987) and (Hale and Wright, 2001). It has been discussed and criticised extensively in recent literature, especially by Dummett (see his (Dummett, 1981) and (Dummett, 1991)). An excellent survey article about Neo-Fregeanism

can be found in (MacBride, 2003). An up-to-date bibiography discussing many different issues of Neo-Fregeanism can be found on the Arche website: http://arche-wiki.st-and.ac.uk/ ahwiki/bin/view/Arche/ MathsBibliographies